Available online at www.sciencedirect.com
ScienceDirect
Computer-Aided Design 39 (2007) 853–862
COMPUTER-AIDED DESIGN
www.elsevier.com/locate/cad

Quasi-developable B-spline surfaces in ship hull design

F. Pérez, J.A. Suárez*

Naval Architecture School of Madrid, Universidad Politécnica de Madrid, (UPM), Avenida Arco de la Victoria SN, 28040 Madrid, Spain

Received 19 September 2006; accepted 26 April 2007

Abstract

The use of developable surfaces in ship design is of engineering importance because they can be easily manufactured without stretching or tearing, or without the use of heat treatment. In some cases, a ship hull can be entirely designed with the use of developable surfaces. In this paper, a method to create a quasi-developable B-spline surface between two limit curves is presented. The centreline, chines and sheer lines of a vessel are modelled as B-spline curves. Between each pair of these boundary curves or directrix lines, the generator lines or rulings are created and a quasi-developable B-spline surface containing the rulings is defined. A procedure based on multiconic development is used to modify the directrix lines in case the rulings intersect inside the boundary curves, avoiding non-developable portions of the surface. B-spline curves and surfaces are widely used today in practically all the design and naval architecture computer programs. Some examples of ship hulls entirely created with developable surfaces are presented.

Keywords: B-splines; Developable surfaces; Shape generation; Ship surface design

1. Introduction

A surface is called a ruled surface if it contains a one-parameter family of straight lines known as generators, or rulings of the ruled surface. In ship design, it is interesting to study the case of ruled surfaces defined by the continuous motion of a straight line between two boundary curves (e.g. centreline profile, chine and sheer line of a hard chine craft according to Fig. 1). A developable surface is a case of ruled surface defined by nonintersecting generators with the same tangent plane at all points of the same generator.

This developable surface will have the property that by successive small rotations around each of the rulings, the surface can be laid flat or developed onto a plane without stretching or tearing. Conversely, a sheet material (e.g. plywood or aluminium) can be shaped into a developable surface with only simple unidirectional bending along the generating lines. These surfaces are also known as singly curved surfaces (Fig. 2, left), since one of their principal curvatures is zero, and therefore, the Gaussian curvature that is the product of both curvatures of the surface is zero.

Doubly-curved surfaces (Fig. 2, right) in hull plating should be avoided, although this is not always possible. A doubly-curved plate will usually require heat treatment and increased work input to achieve its required shape. Single-curvature plates also lead to less scrap. In any case, curvatures of plates should be kept small enough to avoid castings as these make the structural detail three to four times more expensive. The use of single-curvature plates improves welding productivity by facilitating the use of automatic welding machines. Both frame/plate and plate/plate connections are easier to weld than in curved contours.

Apart from the classical developable surfaces, cones and cylinders, the generic case for developable surfaces is a tangent surface of a space curve. Any developable surface is the envelope of a one-parametric family of planes. These planes will be tangent along a line contained in the surface and these lines are the generators or rulings.

In this paper, a method to generate quasi-developable surfaces with B-spline surfaces is presented and applied to ship design. The approximation of obtaining the rulings between a pair of directrices is used and these two curves will be modelled with B-splines. This is detailed in Sections 4 and 5. The way to avoid non-developable areas is made with the substitution of the regression area (if present) with a multi-conic technique that will ensure a developable surface replacing the regression

* Corresponding author.
E-mail address: jose.a.suarez@siport21.com (J.A. Suárez).

0010-4485/$ - see front matter
doi:10.1016/j.cad.2007.04.004

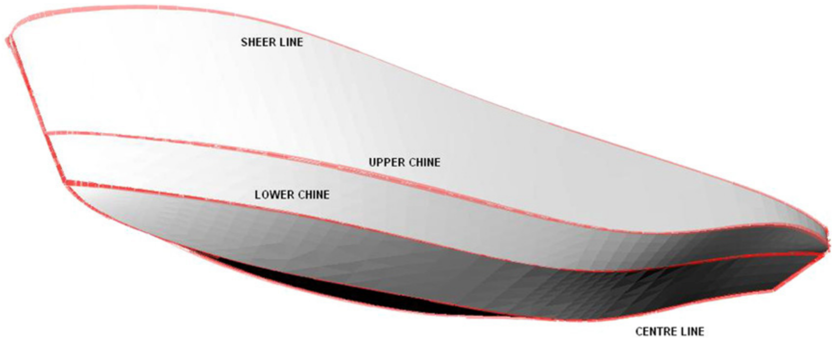

Fig. 1. Boundary curves or directrices of a ship hull.

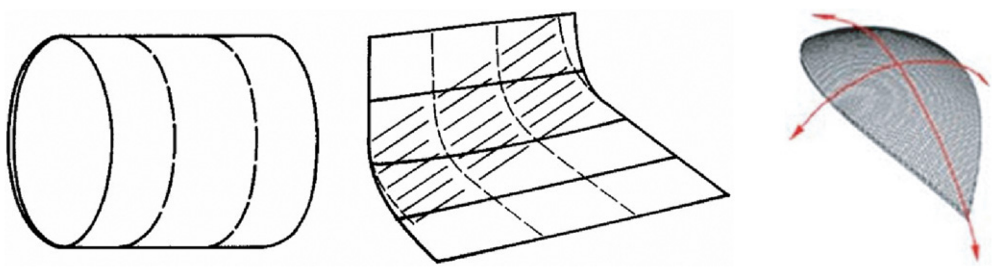

Fig. 2. Single and double curvature.

area as explained in Section 6. Once the rulings are obtained, a B-spline surface that will contain them is created in Section 7. By repeating the procedure between the different 3D lines of the ship (Fig. 1), these kinds of chine hulls can be entirely modelled with the use of developable B-spline surfaces as will be presented in the examples. The presented method cannot create exact developable surfaces with zero Gaussian curvature, but quasi-developable surfaces are used in normal shipyard practice as explained in Section 8. Originality of this work is described in the next section.

2. Background

Developable surfaces are a research field in Geometric Design and several authors have produced interesting ideas for marine design. Reference [1] defines a developable Bézier surface in terms of two Bézier curves (directrix lines or directrices) and rulings between pairs of points from each curve. Both directrices are restricted to parallel planes, making the tangent vectors of each ruling parallel. An algorithm for producing general Bézier developable patches is shown in [2]. This algorithm, based on affine applications of the cells of the control net of the patch, is further developed in [3] by use of degree elevation. Reference [4] extended the procedure of [1] using several Bézier developable surfaces strung together by joining them along their end rulings in C^2 continuity (second derivatives). Properties of developable Bézier patches from two boundary curves are also studied in [5]. Reference [6] obtained conditions for the control nets and weights of a NURBS surface in order to be developable, without the technique of obtaining the rulings between two directrices, which leads to a complicated nonlinear system of conditions that is hard to use for the average designer. Reference [7] used a new representation for developable surfaces in terms of plane geometry using the duality between points and tangent planes in 3D projective space. They interpret the developable surface as the set of its tangent planes represented in the dual form. These techniques are also used in [8] for approximating developable surfaces.

In the marine field, [9,10] applied the design of developable surfaces that rest on two directrices (chines, centreline or sheer line) of important application in small craft design. They used the same approximation as previously mentioned in [1], that is the search of ruling lines between a pair of curves. In fact, they define the surface by their rulings and they used Theilheimer splines to represent the directrices.

Reference [11] used B-spline surfaces, but they started from one directrix curve and a pair of rulings. The second directrix is created between the end points of the rulings assuring developable surface with the use of certain constraints.

From the design point of view, it is more interesting to work with the classical two directrices since the kind of ships that one can design with developable surfaces (hard chine crafts) are designed this way and their hydrodynamic and stability properties are a function of the directrices (chines, centreline). The problem arises when parts of the surface where the rulings intersect each other inside the surface (regression area) are not developable, which can be solved with a good design of chine or sheer lines, or with latter modifications of these lines, as will be seen.

Reference [12] used the same design principles as [11] with the use of a normal directrix to the surface and Catmull-Rom and Beta-splines to model the curves. Reference [13]

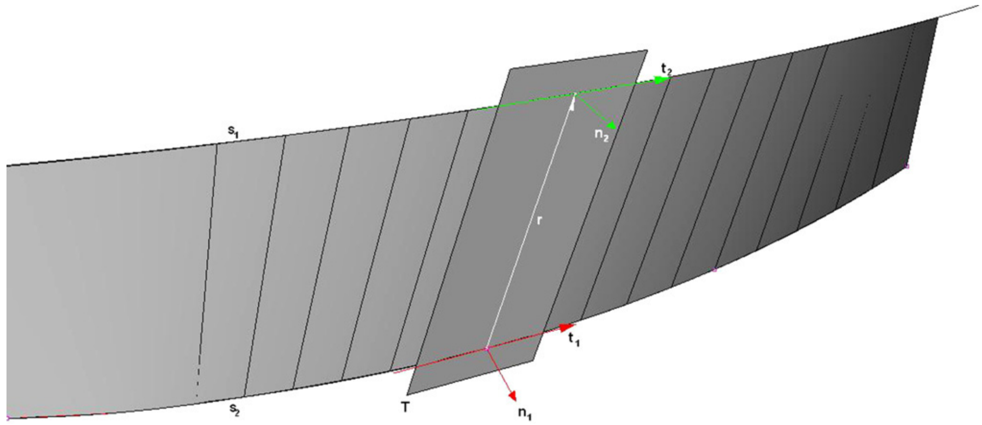

Fig. 3. A ruling of a developable surface.

uses conical and cylindrical surfaces in shipyard applications. Finally, a review of the different techniques for constructing developable surfaces is made in [14].

The originality of this work is the use of B-spline properties for the search technique of the rulings based on a minimum warp angle. The directrices do not have to present any special features. Extension in the ends of the directrices is also possible with this method and some problems to define the limits of the developable surface can be solved. The method is fast and easy enough to be used in a shipyard technical office and it is validated with two small prototypes presented in Section 9.

3. Finding a developable surface

In our case we will search for the different tangent planes, T, to the surface that will also be tangent to the directrix lines $\mathbf{s_1}$ and $\mathbf{s_2}$ (see Fig. 3). These planes will be tangent along a line r contained in the surface and these lines are the generators or rulings. As a consequence, the normal vectors $\mathbf{n_1}$ and $\mathbf{n_2}$ at the endpoints of a ruling will be parallel.

The conditions that the vector of a ruling $\mathbf{r}$ must satisfy can be described vectorially and can be expressed from the definition of the cross-product:

$$\begin{aligned}\mathbf{n_1} &= \mathbf{r} \times \mathbf{t_1}\\ \mathbf{n_2} &= \mathbf{r} \times \mathbf{t_2}.\end{aligned} \tag{1}$$

If $\mathbf{n_1}$ and $\mathbf{n_2}$ are parallel, $\mathbf{r}$ will be a ruling of the surface and this is satisfied when:

$$\mathbf{n_1} \times \mathbf{n_2} = \mathbf{0}. \tag{2}$$

The module or length of the vector in Eq. (2) can be written as $|\mathbf{n_1} \times \mathbf{n_2}| = |\mathbf{n_1}| \cdot |\mathbf{n_2}| \cdot \sin(\mathbf{\Phi})$ and working with unitary vectors, $|\mathbf{n_1} \times \mathbf{n_2}| = \sin(\mathbf{\Phi})$, where $\mathbf{\Phi}$ is called the warp angle. This angle has a physical significance that is of interest. It is the angle that the tangent plane must warp in order to be tangent to both directrix lines. In shipyard practice, small warp angles are permitted, since this is a function of the material and of its thickness as will be explained in Section 8.

4. Working with B-spline curves and nomenclature

As previously mentioned, the directrices $\mathbf{s_1}$ and $\mathbf{s_2}$ will be modelled as B-splines. Therefore the first step is to model the chines, centre line and sheer lines as B-splines. A B-spline curve of degree n with $N+1$ control points is a parametric curve in u with the general form of Eq. (3):

$$\begin{aligned}\mathbf{s}(u) &= \sum_{i=0}^{N} \mathbf{V_i} \cdot {B_i}^n(u) = (X(u), Y(u), Z(u))\\ &= \sum_{i=0}^{N} \left(X_i \cdot B_i^n(u), Y_i \cdot B_i^n(u), Z_i \cdot B_i^n(u)\right)\end{aligned} \tag{3}$$

where $\mathbf{V_i} = (X_i, Y_i, Z_i)$ $(i = 0, \dots, N)$ are the control points of the B-spline and $B_i^n(u)$ are basis functions that depend of a list of knots $\{u_{-1}, \dots, u_{N+n}\}$ with $u_j \leqslant u_{j+1}$. In this paper the uniform parametrization is used and cubic B-splines $(n = 3)$ are considered.

The derivatives $\mathbf{s}'(u)$ of a B-spline $\mathbf{s}(u)$, at any value of the parameter u can be obtained in a recursively way according to Eq. (4)

$$\begin{aligned}&[B_j^n(u)]' = n \cdot \left[\frac{B_{j-1}^{n-1}(u)}{u_{j+n-1} - u_{j-1}} - \frac{B_j^{n-1}(u)}{u_{j+n} - u_j}\right]\\ &\mathbf{s}'(u) = \sum_{i=0}^{N} \mathbf{V_i} \cdot [B_i^n(u)]'\\ &\quad = \sum_{i=0}^{N} \left(X_i \cdot [B_i^n(u)]', Y_i \cdot [B_i^n(u)]', Z_i \cdot [B_i^n(u)]'\right).\end{aligned} \tag{4}$$

In the examples of this work, the authors have used [16] to approximate the different points of the directrices with cubic B-splines $\mathbf{s_1}(u)$ and $\mathbf{s_2}(u)$.

The designer should use a minimum number of control points to model the directrices, in order to obtain faired curves. Nevertheless, numerical fairing algorithms such as [17] or [18] may be applied. These methods for spline curves are usually grounded on knot removal procedures and can alter the form of the splines, reducing precision in the approximation of the data points.

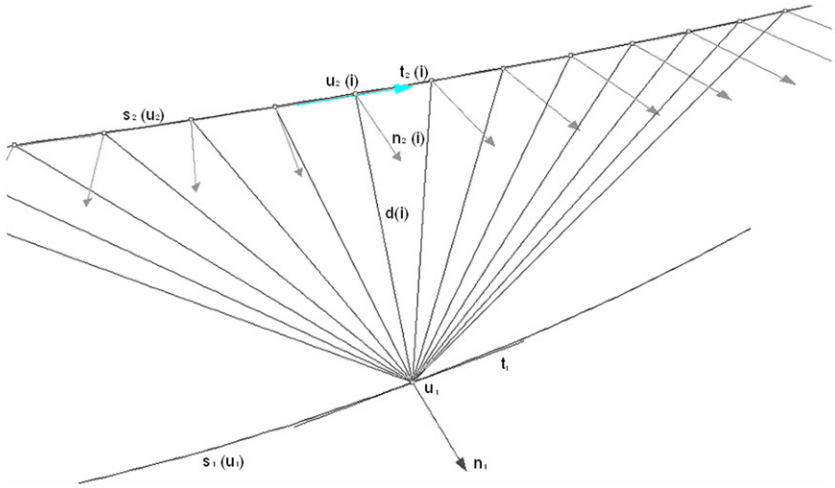

Fig. 4. Searching for the rulings.

If the directrices are fair, the locus of rulings defining the developable surface between them will also be fair and continuous, since the rulings reflect the character of the curves they span. In the case of modelling with developable surfaces, the problem of fitting a fair surface is reduced to find a fair representation for the directrices.

5. Searching for the rulings

Although the expression for a ruling is simple in terms of vectors, it leads to a very complex and awkward expression when reduced to a function of the parameters u_1 and u_2 belonging to the directrices $\mathbf{s_1}(u_1)$ and $\mathbf{s_2}(u_2)$ modelled as parametric B-splines [6]. Multiple roots can also be possible depending on the form of the directrices and the range of variation of the parameters. A numerical approach has been chosen.

For a fixed value of the parameter u_1 on the directrix $\mathbf{s_1}$, which will be one of the ends of the ruling that we search, the corresponding point coordinates and tangent value can be calculated with Eqs. (3) and (4). Different values of the parameter u_2 on the second directrix $\mathbf{s_2}$ are obtained with a step size h (Fig. 4). For different values of u_2 along $\mathbf{s_2}$, generic i points are obtained and its tangent $\mathbf{t_2}(i)$ on $\mathbf{s_2}$ can be computed using Eqs. (3) and (4), the vector $\mathbf{r}(i)$ is easily obtained and with this vector the value of $\mathbf{n_2}(i)$ can be calculated with Eq. (1). A schematic flowchart is depicted in Table 1.

The values for u_2 lie inside the interval $[u_{-1}, \ldots, u_{N+n}]$ that is the list of knots that has been used to construct the B-spline basis and this is normally [0,1] for the sake of simplicity but it is not relevant for the calculations. When the first end of the ruling, u_1 lies near one of the ends of the first directrix, it is possible that no solutions are found for u_2 inside [0,1] and the search has to be extended to a bigger interval, i.e. $[-0.1,1.1]$. B-splines are made of pieces of polynomials, and the extension of the interval outside the list of knots means an extrapolation using the first piece of the B-spline for values in $[-0.1,0]$ and the last piece for values $[1,1.1]$. The limit values -0.1 and 1.1 work well for the different examples in which the algorithm has been tested.

The search will stop for the ith value of u_2 that makes the normal vector parallel, or the cross-product equal to zero according to Eq. (2). As u_2 moves along the second directrix $\mathbf{s_2}(u)$, an example of the variation of $|\mathbf{n_1} \times \mathbf{n_2}| = \sin(\boldsymbol{\Phi})$ can be seen in Fig. 5. When $\sin(\boldsymbol{\Phi})$ presents a minimum value, a local search begins by reducing the step size h and a new set of $u_2(i)$ values is obtained.

Table 1
Schematic algorithm of the method

- **Initial search of a ruling**

First calculate one on the ends of the ruling $\mathbf{Q} = \mathbf{s_1}(u_1)$ in $\mathbf{s_1}$ and compute $\mathbf{t_1}$

For $u = u_1$ to u_{N+n}
    $i = i + 1$
    $u_2(i) = u$
    Calculate $\mathbf{P} = \mathbf{s_2}(u)$
    Calculate $\mathbf{r}(i) = \mathbf{PQ}$
    Calculate $\mathbf{t}_2(i)$ with Eq. (4)
    Calculate $\mathbf{n_1}(i)$ and $\mathbf{n_2}(i)$ Eq. (1)
    Calculate the warp angle $\Phi(i)$
Next u

- **Local search**

A minimum value of the warp angle is detected at any $u_2(i)$? It is bellow the tolerance for the material? Then use a local search strategy repeating the steps of the initial search but enclose $u \in [u_2(i-1), u_2(i+1)]$ until the warp angle is below a tolerance or is low enough for the construction point of view. The local search can be made with Newton method, decreasing step size h or other numerical method.

- Repeat the search for different values of u_1 along $\mathbf{s_1}$ to obtain different rulings. Then a lofting surface that contains the rulings can be constructed according to Section 7
- Consider the searching limits to avoid multiple solutions.
- Detect crossing between rulings that will produce an area of regression, which can be solved following Section 6 or redesigning the shape of the directrices.
- Study the searching limits when calculating rulings at the end points of the directrices.
- If a minimum value for the warp angle cannot be found below the material tolerance, changes in the design should be made.

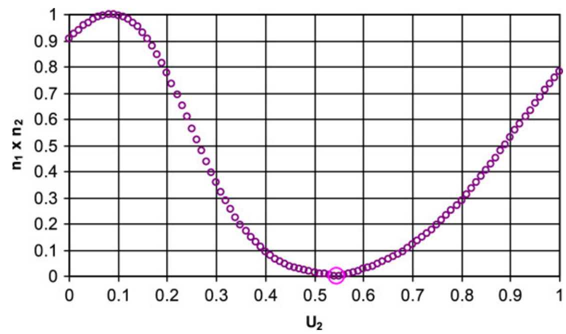

Fig. 5. Variation of $|\mathbf{n_1} \times \mathbf{n_2}|$.

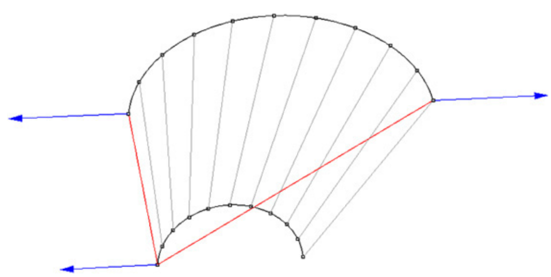

Fig. 6. More than one possible solution.

The search strategy continues until the cross product is under a tolerance, that is a function of the material and its thickness as will be seen in Section 8. When this value is reached, the second end of the ruling is found. More than one solution for the second end of a ruling can be found, depending on the form of the directrices. These values can be eliminated simply by limiting the range of variation of u_2 to an interval where the right solutions are placed. An example of the trunk of a cone is shown in Fig. 6. A point on one lower directrix of this figure can have more than one correspondent solution in the upper one, with a parallel normal vector. So, the ruling at the left of the figure is the correct one and limiting the u_2 search to this area will avoid wrong solutions.

By selecting different u_1 values along the first directrix and repeating the searching strategy that has been described for the second end of the rulings, the rulings of the developable surface that rests on the two directrices can be founded. An example is shown in Fig. 7. The control points of the B-splines can be seen in this figure, 5 points in this case. As mentioned, when u_1 is selected near one of the ends of the directrix, u_2 may lay out of the limits of the list of knots, and the second end of some rulings can be placed in the extension of the second directrix. Once the surface is defined, it can be trimmed away.

It is important to note that the rulings can be also studied as B-splines curves, with degree $n = 1$ and two control points, $N = 1$, placed at the ends of the segments. This will facilitate the creation of a lofting B-spline surface that contains the rulings, as will be explained in Section 7. The correct definition of the rulings is important in shipyard practice because they are used to form the surface. Some devices can be created in practice, such as the "ruling jig" from reference [19] to conform the surface from a flat plate where the rulings and developed directrices have been marked, Fig. 8.

6. The area of regression

Given two directrices $\mathbf{s_1}$ and $\mathbf{s_2}$ the possibility of constructing a developable surface containing both curves is the first step. Many ruled surfaces containing both curves may exist, but no developable surface may be possible. No simple theorem is available to provide an easy test. In design practice, if in each projection of $\mathbf{s_1}$ and $\mathbf{s_2}$ in the profile and plan views, their curvatures have constantly the same sign and pronounced curvature is avoided in these curves, a developable surface will enclose them.

If the rulings are obtained according to Section 5, and none of the rulings intersect between $\mathbf{s_1}$ and $\mathbf{s_2}$, then the ruled surface defined by these generators and bounded by $\mathbf{s_1}$ and $\mathbf{s_2}$ is developable. If some rulings overlap between $\mathbf{s_1}$ and $\mathbf{s_2}$ as in Fig. 9 left, then the edge of regression of the ruled surface has crossed one of the directrix curves and the surface is not locally developable in this area.

If these zones are present, they can be converted into developable areas with the use of a multiconic development, that will modify one of the directrix curves, $\mathbf{s_2}$, between two rulings L_1^* and L_R^* (Fig. 9 left). The new area will be constructed by several rulings belonging to different conic surfaces (developable) that lean on the directrix that is not modified. This technique is a well known procedure of Descriptive Geometry, and the presented algorithm applies the B-splines properties.

With the use of the u parametric B-splines, the area of regression is limited in one of the directrices inside the interval $[u_{11}, u_{1R}]$ and in the second directrix by $[u_{21}, u_{2R}]$. So, L_1^* : $\overline{\mathbf{P_1Q_1}}$ and L_R^* : $\overline{\mathbf{P_RQ_R}}$ are the limit rulings that set the part of the surface where some rulings cross others. By using the

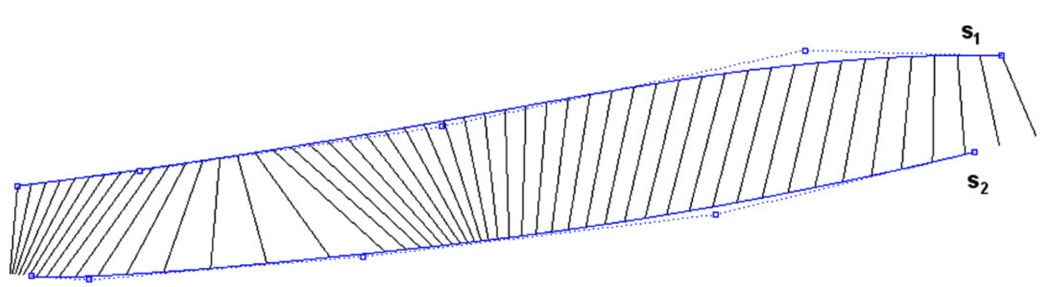

Fig. 7. Rulings of a developable surface.

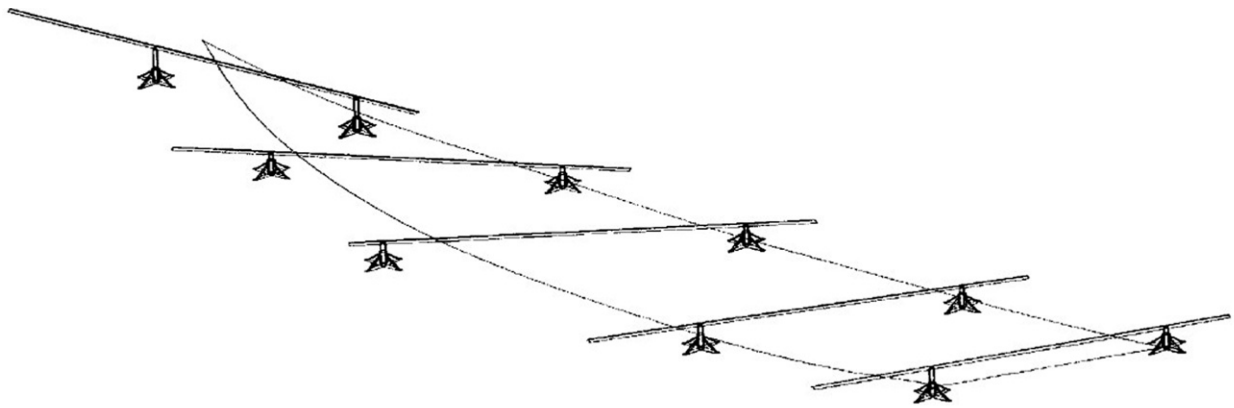

Fig. 8. Ruling jig, from reference [19].

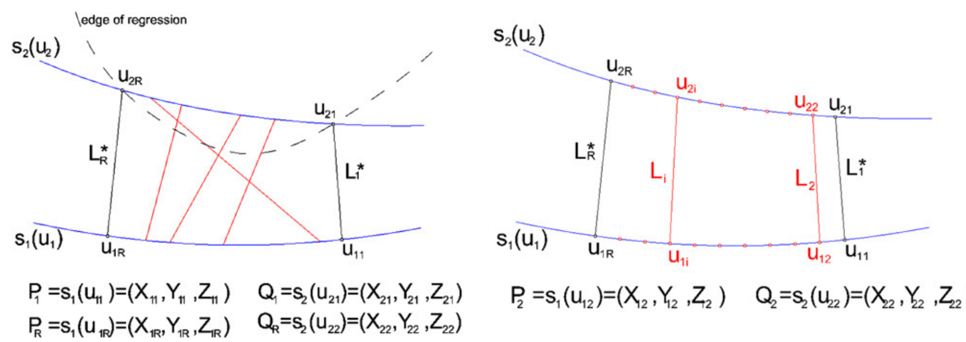

Fig. 9. Area of regression.

search technique described in Section 5, the parameter values that set the ends of a ruling have to be greater than the values of the previous ruling line. If not, the ruling will intersect the previous one and this part of the surface will be an area of the regression.

The multiconic development starts subdividing in equal R parts the arcs limited by [$\mathbf{P_1}$, $\mathbf{P_R}$] and [$\mathbf{Q_1}$, $\mathbf{Q_R}$]. With the use of B-splines, the length of an arc between $u = a$ and $u = b$ can be obtained with the use of Eq. (5)

$$l = \int_a^b \sqrt{X'(u)^2 + Y'(u)^2 + Z'(u)^2}\mathrm{d}u. \qquad (5)$$

The value of the derivatives is calculated with the use of Eq. (4). L_2 is the line that joins the correspondent subdivisions next to L_1^* as in Fig. 9 right. Notice that this segment is not a ruling of the surface.

The multiconic algorithm will find a cone that contains points $\mathbf{P}_2$ and $\mathbf{P_1}$, and a generatrix in L_1^* and will search for a second generatrix L_2^* through $\mathbf{P}_2$. In this way, the portion limited by L_1^*, L_2^* and $\mathbf{P_1}$, $\mathbf{P}_2$ will be a part of a cone and can be developed. Repeating the process with L_1^* and L_R^*, the area of regression will be substituted by a multiconic developable area, with rulings that do not intersect inside the surface limits.

In this case, one of the directrix $\mathbf{s_1}$ will be maintained, whilst the other one will be slightly changed to contain new points placed in the generatrix of the different cones that will be constructed. This technique is graphically described in Fig. 10, left. Starting with $\mathbf{P}_2$ and $\mathbf{P_1}$, Fig. 10 left, a plane α that contains L_2 and perpendicular to the one defined by T, tangent vector in $\mathbf{Q}_2$ defined according to Eq. (4), and L_2 is defined. The intersection of L_1^* and α is calculated. This point $\mathbf{A_1^*}$ will be the vertex of the cone that we are looking for. The line that goes from $\mathbf{A_1^*}$ through $\mathbf{P}_2$ is another generatrix and will contain the new ruling.

A plane β that contains T and $\mathbf{j}$, the unitary vector parallel to the y axis is constructed. The intersection between β and the line from $\mathbf{A_1^*}$ through $\mathbf{P}_2$ is $\mathbf{Q}_{2,}^*$ the second end of the ruling that will substitute to $\mathbf{Q}_2$. The line $L_2^* : \overline{\mathbf{P}_2\mathbf{Q}_2^*}$ is a valid ruling. Repeating this process for consecutive segments L_i and L_{i-1}^* and obtaining the cone vertex $\mathbf{A}_i^*$ and the point $\mathbf{Q_i^*}$ as described, the regression area with crossing rulings is changed by a multiconic surface that is developable. So, the segment between $\mathbf{P_i}$ and $\mathbf{Q_i^*}$ is a valid ruling. The values for $\mathbf{A}_i^*$and $\mathbf{Q_i^*}$ can be calculated with Eq. (9), described in the Appendix (see also Box I).

From the design point of view, it is better to avoid pronounced curvature when designing the chine or the sheer line and maintain their curvatures constantly with same sign, in order to avoid regression areas. This way, the designer will have total control of the shape of the directrices, which are modified if the multiconic algorithm is used. The tolerance for the warp angle used in the calculation of the rulings has an influence in the reduction of the regression area, if this region appears. It is explained in Section 8.

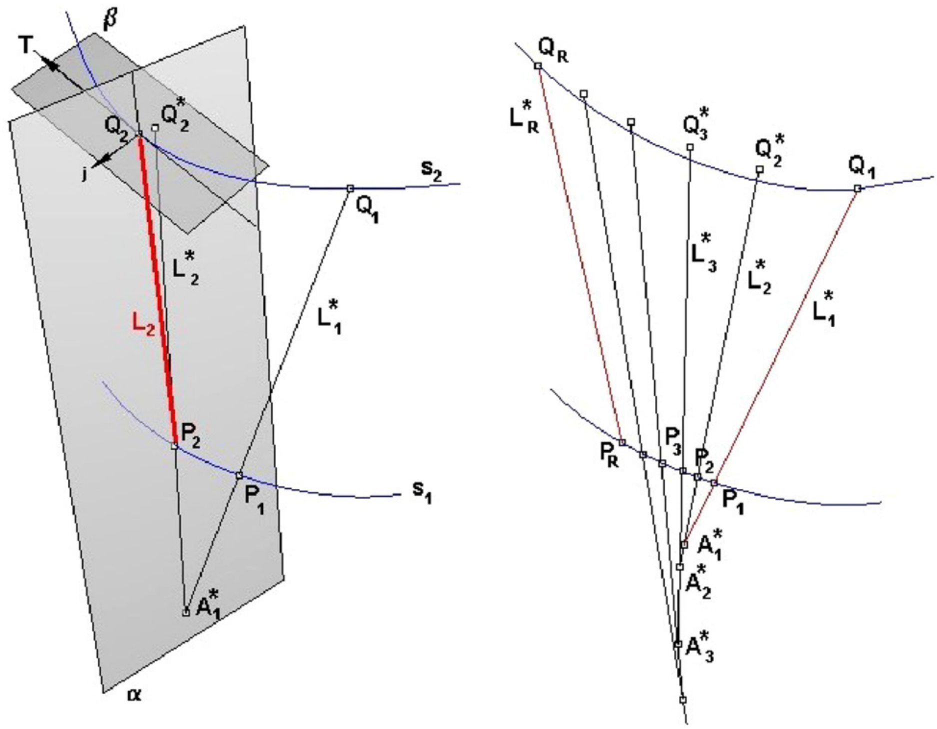

Fig. 10. Multiconic development.

7. Generation of a B-spline surface through the rulings

Up to this point, the rulings have been settled according the previous sections, but the generalization from curves to surfaces is not difficult thanks to the properties of B-splines, and a lofting surface of the rulings can be worked out.

The transition from spline curves to spline surfaces is simple, by just turning the control polygon into a control net of control points $\mathbf{W_{ij}}(X_{\mathbf{ij}}, Y_{\mathbf{ij}}, Z_{\mathbf{ij}})$, and with the use of the same B-spline basis for the two parameters u and v together with the use of two different lists of knots $\{u_{-1}, \ldots, u_{N+n}\}$, $\{v_{-1}, \ldots, v_{M+m}\}$.

The lofting process of a set of q B-splines with the same degree and list of knots can be formulated as: "find a B-spline surface $\mathbf{S}$ with degree $n \times m$ and $(N+1) \times (M+1)$ control points and list of knots $\{u_{-1}, \ldots, u_{N+n}\}$ and $\{v_{-1}, \ldots, v_{M+m}\}$ according to Eq. (6) that interpolates q different B-splines $\mathbf{c}_d$ $(d = 0, \ldots, q-1)$ of nth degree with $N+1$ control points and list of knots $\{u_{-1}, \ldots, u_{N+n}\}$, with the form of Eq. (7)"

$$\mathbf{S}(u, v) = \sum_{i=0}^{N} \sum_{j=0}^{M} \mathbf{W_{ij}} \cdot B_i^n(u) \cdot B_j^m(v) \tag{6}$$

$$\mathbf{c_d}(u) = \sum_{i=0}^{N} \mathbf{V_{id}} \cdot B_i^n(u) \quad (d = 0, \ldots, q-1). \tag{7}$$

The interpolation can be written:

$$\begin{aligned} \mathbf{S}(u, v_d) &= \sum_{i=0}^{N} \left(\sum_{j=0}^{M} \mathbf{W_{ij}} \cdot B_j^m(v_d) \right) \cdot B_i^n(u) \\ &= \sum_{i=0}^{N} \mathbf{V_{id}} \cdot B_i^n(u) = \mathbf{c_d}(u) \quad (d = 0, \ldots, q-1). \end{aligned} \tag{8}$$

This group of equations has to be solved for a set of values of the parameter v_d $(d = 0, \ldots, q-1)$ that is called the choice of the parametrization. The most usual methods are the uniform parametrization, the parametrization by chord length and the centripetal parametrization, [15]. By identifying equal coefficients for every row of Eq. (8), $i = 0, \ldots N$, the linear system of Eq. (9) is obtained.

$$\sum_{j=0}^{M} \mathbf{W_{ij}} \cdot B_j^m(v_d) = \mathbf{V_{id}} \quad (d = 0, \ldots, q-1). \tag{9}$$

In this particular case $\mathbf{V_{id}}$ are the control points for every ruling d, the ends of each ruling, a surface of degree 1×3 is used with $n = 1, N = 1, m = 3$ and both lists of knots are uniform. In order to obtain a unique solution for Eq. (9), $M + 1 = q$, that is the number of rulings defined according to Section 5. The $(M+1) \bullet (N+1)$ solutions of Eq. (8) are the control points $\mathbf{W}_{ij}$ of the lofting surface of Eq. (6) containing the rulings.

8. Gaussian curvature of the created surfaces

An exact developable surface has zero Gaussian curvature, which is the product of the greatest curvature and the least one. This implies that at least one of the two curvatures is zero and so the surface must contain some straight lines that will be oriented on the directions of the principal curvature (the rulings).

The Gaussian curvature of a B-spline surface can be checked to ensure developability, and this tool is available in all the naval architecture programs that work with NURBS surfaces. The method that has been presented works with certain tolerances in the cross product of the normal vectors of a ruling as previously

Table 2
Parameters of the Example 1

	Sheer	Chine	Centreline
Control Points	(0.00, 0.00, 9.00)	(1.40, 0.00, 5.30)	(1.40, 0.00, 5.30)
	(6.86, 7.10, 8.22)	(10.5, 7.53, 1.93)	(2.26, 0.00, −0.21)
	(21.6, 8.93, 6.25)	(25.7, 7.85, 1.28)	(22.6, 0.00, −0.10)
	(36.9, 8.73, 5.86)	(40.4, 7.46, 1.27)	(36.3, 0.00, −0.10)
	(45.0, 7.65, 6.10)	(44.1, 7.20, 1.70)	(44.1, 0.00, 0.50)
List of knots	(0, 0, 0, 0, .5 1, 1, 1, 1)	(0, 0, 0, 0, .5 1, 1, 1, 1)	(0, 0, 0, 0, .5 1, 1, 1, 1)

mentioned in Section 5 and the Gaussian curvature will be low but not exactly zero.

The form of the generatrices can make that a ruling with a warp angle equal to zero cannot be found, so a certain warp angle may exist in some parts of the quasi-developable surface. Fortunately, almost any material, metal or otherwise, will have sufficient plasticity to accommodate slight deformation and a certain warp angle.

Metal can stand a warp angle in the order of six degrees or so, [19]. Standard strength analysis methods can be used to calculate the tensile strength resulting from a given warp, but a six degree warp generally is approaching the yield of most aluminium alloys of standard thickness. So from a practical point of view, if the warp angle for every ruling is below this limit, the surface can be considered as developable.

Increasing the tolerance for the warp angle reduces the extension of a possible regression area since the surface is stretched until a limit to adapt the original directrix lines. But this will produce internal stresses inside the material that can produce a failure in the structure, so the tolerance should be respected and the multiconic algorithm can be used as explained in Section 6. For the ship examples of the next sections, the developable surfaces that form the ship hulls were constructed in hard paper, a simple sheet material, in order to check the validity of the designed surfaces (Figs. 12 and 14).

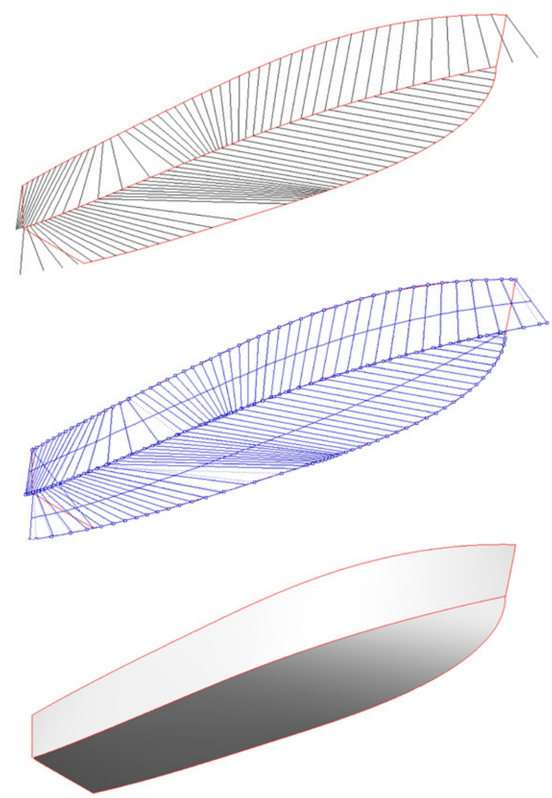

Fig. 11. Rulings, control points and rendered view of the hard chine.

9. Examples

In this section, some examples of developable ship hulls created with the presented method will be shown. These hulls are defined through their chine or chines, sheer and centre lines, and different developable surfaces are created using the mentioned lines as directrices. Once the directrices are defined with B-splines, the rulings are calculated according to Section 5. Then, a B-spline surface that rests upon these rulings can be created according to Section 7. These surfaces can be easily interchanged with specific naval architecture programs with the use of a standard file such as an IGES or a STEP.

9.1. Example 1: Hard chine

This example shows the lines of a hard chine craft. The chine, sheer and centre line can be found in reference [11] and are presented in Table 2.

The hull is modelled with two different surfaces, one between the centreline and the chine, and the second one between the chine and the sheer line. The rulings of both surfaces obtained as mentioned in Section 5 can be seen on Fig. 11, up. Notice that rulings near the ends of the first directrix have their endings in the extension of the second one.

Based on these rulings, two lofting surfaces are created according to Section 7. The control points of both surfaces calculated with Eq. (8) and some isoparametric curves are shown in Fig. 11, center. Once the surfaces are created, they are trimmed away with the centre and transom planes, and the final hull form is obtained, Fig. 11, down. Once the surfaces are created, the Gaussian curvature can be calculated. In this example, the maximum Gaussian curvature obtained is 2×10^{-6}. A small paper prototype is showed in Fig. 12.

9.2. Example 2: UBC fishing vessel

This example shows the lines of a two chine fishing vessel. The geometry of the chines, sheer and centreline is obtained

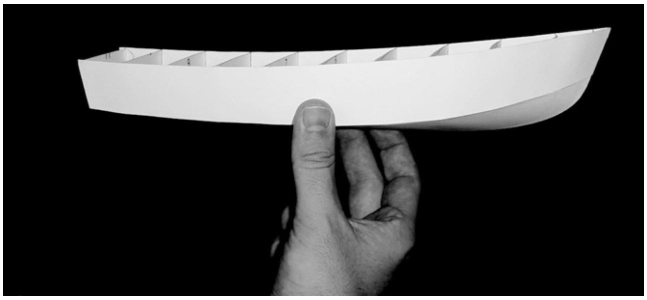

Fig. 12. Paper prototype of the hard chine.

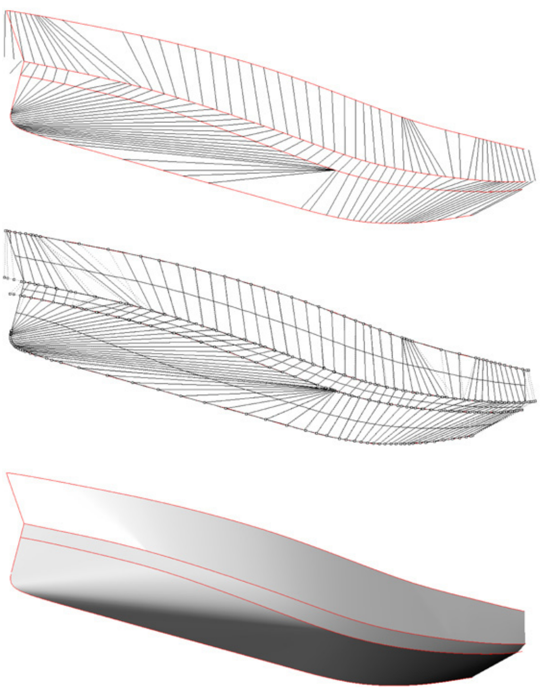

Fig. 13. Rulings, control points and rendered view of the UBC.

from reference [12]. The geometry is more complex than in the previous example, especially in the bow area, but is a good example of the type of lines that can be constructed with the use of developable surfaces. In this case, the hull is constructed with three developable surfaces constructed with the presented method between the upper chine and sheer line, the lower and upper chines, and the centre line and lower chine.

The rulings, the control points of the three surfaces and a rendered view of the trimmed surfaces can be seen in Fig. 13. The maximum Gaussian curvature of this example is 1×10^{-5}. A small paper prototype is shown in Fig. 14.

10. Conclusions

Developable surfaces are of practical interest in shipyard practice. A developable surface will require the minimum strain energy of flexure.

This paper has presented a method to create quasi-developable surfaces that can be considered as developable surfaces in shipyard practice.

Fig. 14. Paper prototype of the UBC fishing vessel.

The rulings are obtained with a searching technique based on the warp angle, and when regression areas are detected, they can be avoided with the use of a multiconic algorithm that will change locally the shape of one of the directrices. If the multiconic algorithm is applied, the error of the approximation can be checked by comparing the maximum deviation between the original and modified directrix.

The creation of a lofting B-spline surface allows the interchange of the created surface with the most common naval architecture programs that work with NURBS surfaces to define the ship hull. This can be made with the use of IGES or STEP standard data exchange.

Two examples have been presented. The UBC series are real ships that are constructed with the use of developable surfaces. The examples are an accurate representation of the kind of ships that can be constructed entirely with the use of developable surfaces. Small prototypes in paper, a simple sheet material, were constructed in order to check the validity of the procedure.

Appendix

Considering the nomenclature of Fig. 9 and dividing the part of the directrices $\mathbf{s_1}$ and $\mathbf{s_2}$ inside the area of regression into $R-1$ equal parts, the position of the vertexes $\mathbf{A}_i^*(X_i^*, Y_i^*, Z_i^*)$, of the cones generated and the new points $\mathbf{Q_i^*}(X_{2i}^*, Y_{2i}^*, Z_{2i}^*)$ that substitute to the points $\mathbf{Q_i}(X_{2i}, Y_{2i}, Z_{2i})$, can be calculated with the following expressions.

The points $\mathbf{Q_i^*}$ together with the points $\mathbf{P_i}(X_{1i}, Y_{1i}, Z_{1i})$ form the ends of the rulings. The derivatives $(X'_{2i}, Y'_{2i}, Z'_{2i})$ at points $\mathbf{Q_i}$ can be calculated with Eq. (4).

$$\gamma_1 = X_{2i} - X_{1i}$$
$$\gamma_2 = Y_{2i} - Z_{1i}$$
$$\gamma_3 = Z_{2i} - Z_{1i}$$
$$\beta_1 = \frac{Y^*_{2(i-1)} - Y_{1(i-1)}}{X^*_{2(i-1)} - X_{1(i-1)}}$$
$$\beta_2 = \frac{Z^*_{2(i-1)} - Z_{1(i-1)}}{X^*_{2(i-1)} - X_{1(i-1)}}$$
$$\beta_3 = \frac{X^*_{i-1} - X_{1(i-1)}}{X^*_{2(i-1)} - X_{1(i-1)}}$$
$$\alpha_1 = \gamma_2 \cdot [\gamma_1 \cdot Y'_{2i} - \gamma_2] + \gamma_3 \cdot [\gamma_1 \cdot Z'_{2i} - \gamma_3]$$
$$\alpha_2 = \gamma_3 \cdot [\gamma_2 \cdot Z'_{2i} - \gamma_3 \cdot Y'_{2i}] - \gamma_1 \cdot [\gamma_1 \cdot Y'_{2i} - \gamma_2]$$
$$\alpha_3 = \gamma_3 \cdot [\gamma_1 \cdot Z'_{2i} - \gamma_3] - \gamma_2 \cdot [\gamma_2 \cdot Z'_{2i} - \gamma_3 \cdot Y'_{2i}]$$

$$X^*_{i-1} = \frac{\alpha_1 \cdot X_{2i} - \alpha_2 \cdot Y_{1(i-1)} + \alpha_2 \cdot X_{1(i-1)} \cdot \beta_1 + \alpha_2 \cdot Y_{2i} - \alpha_3 \cdot Z_{1(i-1)} + \alpha_3 \cdot X_{1(i-1)} \cdot \beta_2 + \alpha_3 \cdot Z_{2i}}{\alpha_1 + \alpha_2 \cdot \beta_1 + \alpha_3 \cdot \beta_2}$$
$$Y^*_{i-1} = Y_{1(i-1)} + \beta_3 \cdot [Y^*_{2(i-1)} - Y_{i(i-1)}]$$
$$Z^*_{i-1} = Z_{1(i-1)} + \beta_3 \cdot [Z^*_{2(i-1)} - Z_{i(i-1)}] \quad i = 2, \ldots, R-1$$

Box I.

See also Box I

$$\delta_1 = \frac{Z_{1i} - Z^*_{i-1}}{X_{1i} - X^*_{i-1}} \qquad \delta_2 = \frac{X^*_{2i} - X^*_{i-1}}{X_{1i} - X^*_{i-1}}$$
$$X^*_{2i} = \frac{Z'_{2i} \cdot X_{2i} + Z^*_{i-1} - X^*_{i-1} \cdot \delta_1 - Z_{2i}}{Z'_{2i} - \delta_1}$$
$$Y^*_{2i} = Y^*_{i-1} + \delta_2 \cdot [Y_{1i} - Y^*_{i-1}] \tag{10}$$
$$Z^*_{2i} = Z^*_{i-1} + \delta_2 \cdot [Z_{1i} - Z^*_{i-1}]$$

References

[1] Aumann G. Interpolation with developable Bézier patches. Computer-Aided Geometric Design 1991;8:409–20.

[2] Aumann G. A simple algorithm for designing developable Bézier surfaces. Computer Aided Geometric Design 2003;20:601–19.

[3] Aumann G. Degree elevation and developable Bézier surfaces. Computer Aided Geometric Design 2004;21:667–70.

[4] Maekawa T, Chalfant JS. Design and tessellation of B-spline developable surfaces. Journal of Mechanical Design 1998;120:453–61.

[5] Chu C-H, Séquin CH. Developable Bézier patches: Properties and design. Computer Aided Design 2002;34:511–27.

[6] Lang J, Roschel O. Developable $(1, n)$ Bézier surfaces. Computer Aided Geometric Design 1992;9:291–8.

[7] Bodduluri RC, Ravani B. Design of developable surfaces using duality between plane and point geometries. Computer-Aided Design 1993;25:621–32.

[8] Pottmann H, Wallner J. Approximation algorithms for developable surfaces. Computer Aided Geometric Design 1999;16:539–56.

[9] Nolan TJ. Computer-Aided design of developable hull surfaces. Marine Technology 1971;4:233–42.

[10] Clements JC. A computer system to derive developable hull surfaces and tables of offsets. Marine Technology 1981;18:227–33.

[11] Chalfant JS, Maekawa T. Design for manufacturing using B-spline developable surfaces. Journal of Ship Research 1998;42:207–15.

[12] Konesky B. Newer Theory and algorithms for computer aided design of developable surfaces. Marine Technology 2005;42:71–8.

[13] Chen H-Y, Lee I-K, Leopoldseder S, Pottmann H, Randrup T, Wallner J. On Surface approximation using developable surfaces. Technical report no. 54. Institute of Geometry, Vienna University of Technology; 2000.

[14] Azariadis P, Sapidis N. Planar development of digital free-form surfaces. In: Sarfraz M, editor. Advances in geometric modeling. Wiley; 2003.

[15] Farin G. Curves and surfaces for CAGD. 5th ed. San Francisco: Morgan Kaufmann; 2001.

[16] Pérez-Arribas F, Suárez-Suárez JA, Fernández-Jambrina L. Automatic surface modelling of a ship hull. Computer-Aided Design 2006;38:584–94.

[17] Farin G, Rein G, Sapidis N, Worsey AJ. Fairing cubic B-spline curves. Computer Aided Geometric Design 1987;4:91–104.

[18] Kjellander JA. Smoothing of bicubic parametric surfaces. Computer Aided Design 1983;15:289–93.

[19] Rolf O, Barry CD, Duffy B, Welter J. Block construction of small ship and boats through use of developable panels. Journal of ship production 2002;18:65–72.

Dr Francisco L. Pérez Arribas is an Associate Professor in the Naval Architecture and Marine Engineering School of Madrid (ETSIN), UPM. He received B.S. and M.S. degrees in Naval Architecture from the Universidad Politécnica de Madrid. He has working experience at the ETSIN Towing Tank in CAD and CASHD. His current research interests include ship hull modelling, parametric ship design and geometric modelling.

José Antonio Suárez is a Ph.D. student at the ETSIN. He received B.S. and M.S. degree in Naval Architecture from the Universidad Politécnica de Madrid. His research activities include parametric ship design and hydrodynamics.