A Density-Management Diagram for Slash Pine Plantations in the Lower Coastal Plain

Thomas J. Dean, *School of Forestry, Wildlife, and Fisheries, Louisiana State University Agriculture Center, Baton Rouge, LA, 70803-6202, and* **Eric J. Jokela,** *Dept. of Forestry, University of Florida, Gainesville, FL 32611-0303.*[1]

*ABSTRACT. Data from 92 regional, midrotation-fertilizer trials were used to develop a density-management diagram for site-prepared slash pine (*Pinus elliottii *var.* elliottii*) plantations. The density-management diagram shows the interrelationships of five important stand variables (i.e., quadratic mean diameter (D_q), trees/ac, site height, standing volume/ac, and relative current annual increment) in a graphical form. The diagram can aid foresters in designing and comparing alternative density-management regimes for slash pine. In doing so, foresters can evaluate individual tree and stand level performances in relation to growing stock levels and make field approximations of growth and yield for various density-management regimes. Results indicated that fertilization and soil type had minimal effects on the diagram's isolines. This suggests broad applicability of the diagram for fertilized or unfertilized plantations found in the lower Coastal Plain. The use of the diagram is illustrated with three alternative density-management regimes, and a method is presented for estimating midrotation fertilization responses.*

South. J. Appl. For. 16(4):178–185

The importance of controlling growing stock levels in southern pine stands is underscored by the numerous spacing and thinning experiments conducted to determine optimal densities for various management objectives. These empirical studies, however, have not provided foresters with adequate tools to devise and modify density-management prescriptions for either new or existing stands. Although Gingrich (1967)-type stocking charts have been developed for southern pines and have been used both to define growing-space requirements and to guide thinning prescriptions (Lorio 1980, Rogers 1983), they have had only limited utility.

More recently, an advanced stocking chart called a *density-management diagram* has been developed (Drew and Flewelling 1979, McCarter and Long 1986). These diagrams represent a graphical display of two concepts: (1) the inverse relationship between tree size and density at maximum stocking; and (2) basic allometric relationships among quadratic mean diameter (D_q), height, and volume. They serve as a valuable field tool and can be used in several ways. For a particular stand, a density-management diagram allows a resource manager to assess stocking or estimate standing volume using two variables. They can also be used to plot stand development over time and to devise alternative density-management regimes to meet certain objectives.

Density-management diagrams have broad applicability because they are largely independent of age and site quality. They are not intended, however, to replace more precise growth and yield models available for a species; growth and yield models are free to incorporate any statistically relevant variable to improve their predictive capabilities. Precision is sacrificed in the diagram for the simplicity of showing the interrelationships of five important variables in graphical form.

Within the United States, the most extensive use of density-management diagrams has been in the West with species such as coastal Douglas-fir (*Pseudotsuga menziesii*), lodgepole pine (*Pinus contorta*) and western redcedar (*Thuja plicata*) (Drew and Flewelling 1979, McCarter and Long 1986, Smith 1989). Their use has been less common in the South (Flewelling 1981). One potential problem in developing density-management diagrams for southern pines is the strong influence that soils have on site quality and the response to cultural treatments (Fisher and Garbett 1980, Bailey et al. 1989). Since the diagram represents a graphical display of several statistical models, it is not known whether separate models are needed for different soil types or various silvicultural treatments such as fertilization.

The objective of this paper is to present a density-management diagram for slash pine (*Pinus elliottii* var. *elliottii*) plantations. Slash pine is planted on a wide range of soil types in the Southeast, and fertilization is a common silvicultural treatment. The second objective is to analyze the effects of soil type and fertilization on the isolines of the diagram. The final objective is to illustrate the range of application and use of the diagram as a management tool.

DIAGRAM CONSTRUCTION

The structure of the density-management diagram presented for slash pine is similar to that developed for lodgepole pine by McCarter and Long (1986). In their diagram, isolines of stocking, standing volume, and site height were plotted as a function of D_q and number of trees/ac (N). This

[1] The authors wish to express their appreciation to S. Stearns-Smith for technical support and to the CRIFF (Cooperative Research in Forest Fertilization) member companies for installing and monitoring the studies. Florida Agric. Exp. Stn. Journal Series Paper No. R-02086.

type of diagram was used because basal area and N are standard mensurational data collected for most stands.

Data for the slash pine density-management diagram were collected from 92 regional fertilizer trials established by the University of Florida's Cooperative Research in Forest Fertilization (CRIFF) program. The experiments were established between 1973 and 1986 and designed to determine fertilizer responses of established slash pine plantations (9 to 20 years old at time of fertilization). Individual tests were installed over a wide range of Lower Coastal Plain site conditions in both thinned and unthinned stands (Figure 1). Soils at each location were classified according to CRIFF soil groups (Fisher and Garbett 1980). With few exceptions, these stands represented converted wildland plantings (i.e., no old-fields). Mechanical site preparation varied among locations but included combinations used by forest industry to meet pulpwood objectives (i.e., shearing, raking, harrowing, bedding, and burning). Likewise, depending on test objectives, the experimental design and fertilizer application rates and combinations were variable among locations.

Current operational fertilization recommendations for established southern pine stands call for a combination of nitrogen and phosphorus applied at elemental rates ranging from 150 to 200 lb/ac and 40 to 50 lb/ac, respectively (Kidder et al. 1987). For our analysis, we selected only a subset of treatment plots (181 unfertilized, 248 fertilized) that included this range. Treated plots were broadcast-fertilized using urea and concentrated superphosphate sources for nitrogen and phosphorus, respectively.

Rectangular measurement plots were about 0.10 ac in size. Treatment plots extended two planting rows beyond the measurement plots and included at least two additional rows of untreated buffer. At the time of treatment, all trees were tagged and measured for diameter at breast height (dbh, 4.5

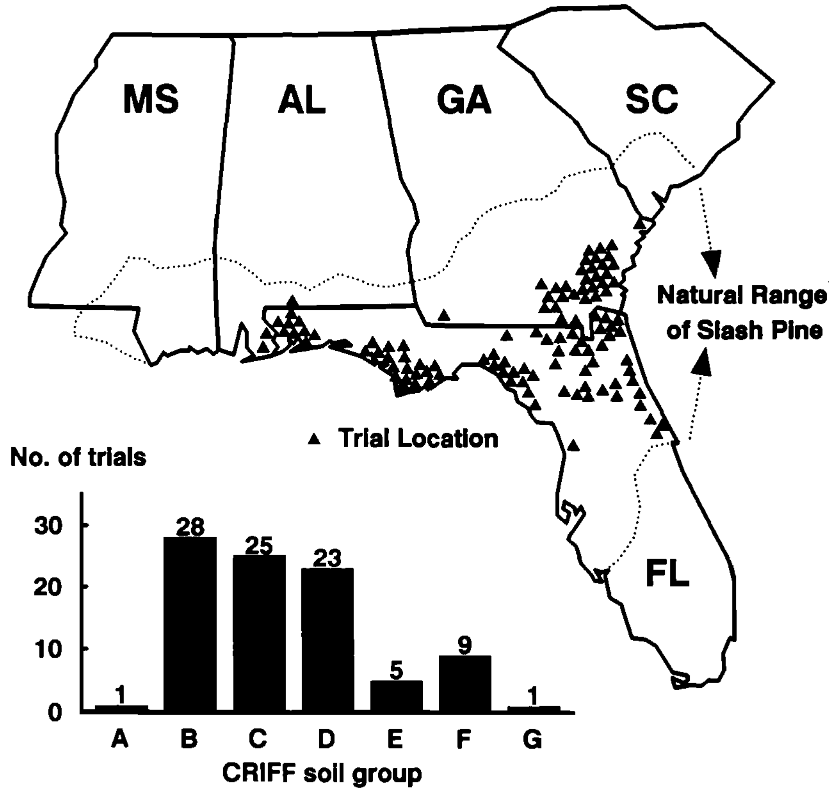

Figure 1. Distribution of test installations by geographic location and CRIFF soil group.

ft) and total height. Plots were remeasured approximately every 2 or 3 years, depending on the test, for up to 10 years after treatment. Standing volume (total, outside bark—6 in. stump) was calculated for all trees ≥ 3 in. using a generalized equation developed by Bailey et al. (1982). Ranges in mensurational data for the test installations are shown in Table 1.

Data from the fertilized and unfertilized plantations were randomly assigned into two groups. One group was used for curve fitting while the other was used for model verification. The following equations were fitted to the first group of data using nonlinear regression techniques (McCarter and Long 1986):

$$D_q = (\beta_0 + \beta_1 \cdot \overline{V})^{\beta_2} \cdot (1 - \beta_3 \cdot N)^{\beta_4} \quad (1)$$

$$\overline{V} = (\theta_0 + \theta_1 \cdot D_q^{\theta_2}) \cdot (\theta_3 \cdot H_s^{\theta_4}) \quad (2)$$

where $\beta_0 - \beta_4$ and $\theta_0 - \theta_4$ are regression coefficients; D_q is quadratic mean diameter; $\overline{V}$ is mean stem volume; and H_s is site height, i.e., the average height of the largest (in dbh) 52% of the trees in the plot (R.L. Bailey, personal communication). The form of the equations is necessary to fit the isolines for standing volume and site height. Isolines for standing volume were developed using Eq. (1) by setting $\overline{V}$ constant and solving for D_q through a range of N. Isolines for site height were determined by an iterative procedure using both Eqs. (1) and (2). Site

Table 1. Range in quadratic mean diameter (D_q, in.), trees/ac (N), basal area (BA, ft²/ac), standing volume (V, ft³/ac), site height (H_s, ft), and age (yr) for the unfertilized and fertilized slash pine plantations used to develop the density-management diagram.

Treatment	D_q	N	BA	V	H_s	*Age*
Unfertilized	2.0–8.8	126–752	13–147	5–3905	14.0–73.8	9–28
Fertilized[1]	2.2–8.9	126–892	16–166	3–4204	14.1–73.7	9–28

[1] Average 4-year fertilizer response (ft³ ac⁻¹ yr⁻¹) by CRIFF soil group: A = 145.1; B = 51.7; C = 53.7; D = 67.1; E = 30.1; F = 37.1; G = −3.0; Overall average = 54.0.

height is held constant as N is incremented across a range of values. At each value of N, D_q is found such that values of $\bar{V}$ in Eqs. (1) and (2) are equal. Summary statistics for the D_q and $\bar{V}$ curve fits are shown in Table 2. Residuals from the verification data set were unbiased with respect to the independent variables and site index.

In McCarter and Long's (1986) diagram, stocking was expressed in terms of Reineke's stand density index (SDI). Since doubts remain concerning whether SDI applies to slash pine (Reineke 1933), stocking was expressed in terms of percentage of current annual growth potential. The equation used for the isolines in this study was

$$D_q = [(15140.74 \cdot P)/(100 \cdot N)]^{0.662} \quad (3)$$

where P = percent of maximum current annual growth potential. The value of P was determined by the formula

$$P = I_n/\max(I_n) \cdot 100 \quad (4)$$

where I_n = normalized gross volume increment (i.e., annual gross-volume increment per annual height increment of the dominant and codominant trees; T.J. Dean and E.J. Jokela, 1991, unpublished data). The basis for I_n has been described by Arney (1985). The value of I_n was related to D_q and N by the nonlinear regression equation

$$I_n = 0.0135 \cdot D_q^{1.51} \cdot N \quad (5)$$

The maximum value of I_n represented the highest predicted value from Eq. (5) using observed values of D_q and N in the sample plots.

THE DIAGRAM

The density-management diagram for slash pine is shown in Figure 2. Isolines for current annual growth potential, standing volume, and site height ranged from 20 to 100%, 50 to 8000 ft³/ac, and 10 to 100 ft, respectively. The range of these values approximate the original data set (cf. Table 1) and correspond to those typically reported in yield tables for slash pine plantations (e.g., Clutter and Belcher 1978, Dell et al. 1979, Schroeder et al. 1979).

Table 2. Nonlinear regression coefficients for models predicting quadratic mean diameter (D_q) and mean stem volume ($\bar{V}$) for unfertilized and fertilized slash pine plantations.[1]

$D_q = (\beta_0 + \beta_1 \cdot \bar{V})^{\beta_2} \cdot (1.0 - \beta_3 \cdot N)^{\beta_4}$

Treatment	β_0	β_1	β_2	β_3	β_4	r^2
Unfertilized	9.329	119.557	0.2944	$1.074 \cdot 10^{-4}$	1.1726	0.97
Fertilized	7.119	142.321	0.2881	$1.702 \cdot 10^{-4}$	0.7497	0.97

$\bar{V} = (\Theta_0 + \Theta_1 \cdot D_q^{\Theta_2}) \cdot (\Theta_3 \cdot H_s^{\Theta_4})$

Treatment	Θ_0	Θ_1	Θ_2	Θ_3	Θ_4	r^2
Unfertilized	0.0045	0.0012	2.181	1.359	0.988	0.98
Fertilized	0.0050	0.0017	2.157	1.084	0.991	0.98

[1] N = trees/ac; H_s = site height; $r^2 = (1 - ESS/TSS)$ where ESS = error sum of squares and TSS = total sum of squares.

The mean residuals for the verification data stratified by CRIFF soil group were not significantly different from zero in most cases.

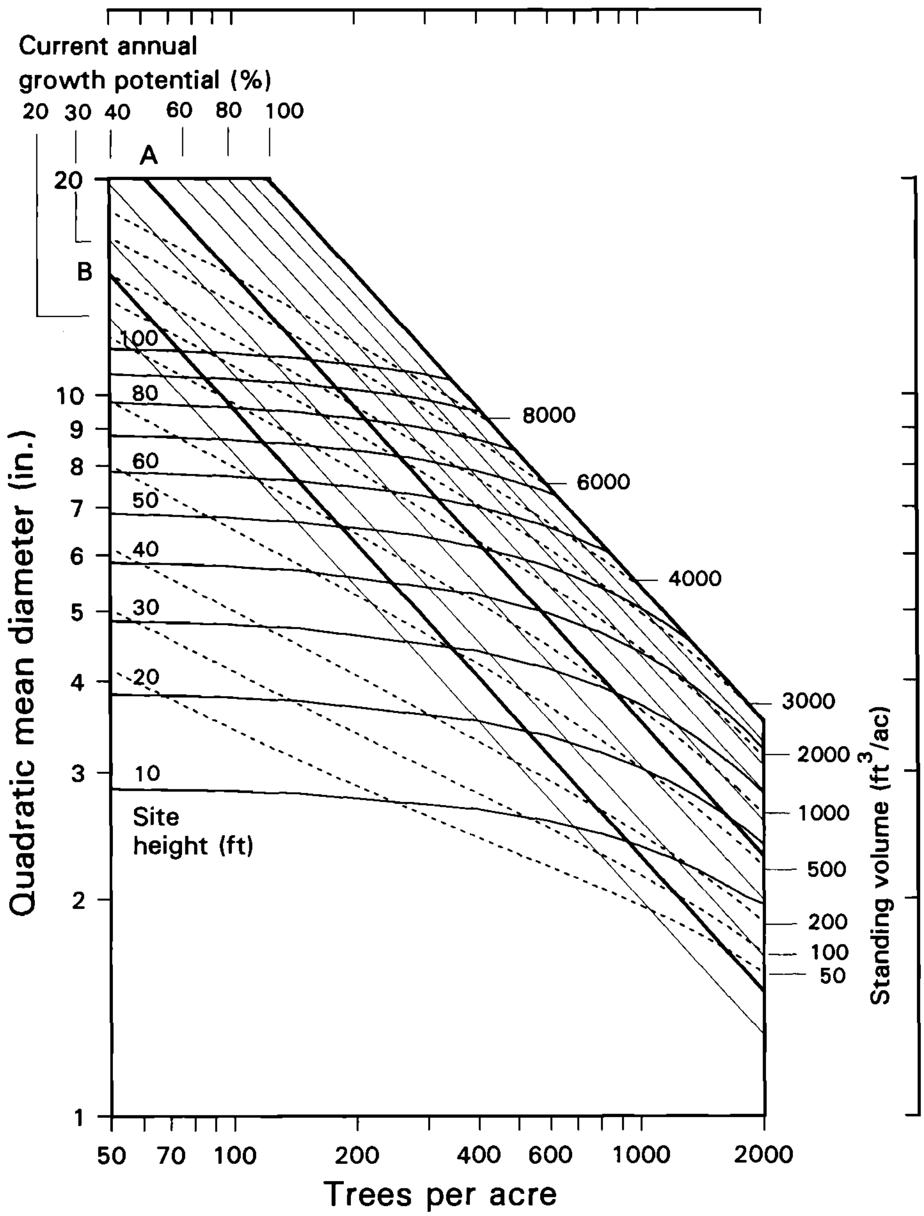

Figure 2. A density-management diagram for slash pine plantations in the lower Coastal Plain

This indicates that Eqs. (1) and (2) are unbiased with respect to CRIFF soil groups B–F (Table 3). Exceptions occurred for fertilized plantations located on very poorly drained savanna (CRIFF A) and excessively drained sandhill soils (CRIFF G). For these extremes in soil drainage, the mean residuals were significantly different from zero and up to 9% greater than the mean values of D_q and $\bar{V}$. This bias probably reflects the small sample size available (Figure 1), and caution is therefore required when applying the diagram to plantations on these soils.

Fertilization with nitrogen and phosphorus significantly increased cubic foot volume yields of the sample plantations (Table 1), averaging 54 ft^3/ac/yr across all soil groups after 4 years. Fertilizer additions, however, had a negligible effect on the position and shape of the isolines for total standing volume and site height (Figure 3). This indicates that a separate density-management diagram for fertilized slash pine plantations is not needed. Previous research with slash pine has similarly demonstrated that fertilization can significantly accelerate stand development, yet it only has a relatively minor effect on volume-dimensional relationships and yield predictions (Jokela et al. 1989, Colbert et al. 1990).

The interrelationships among D_q, N, site height, and total standing volume for the density-management diagram compared favorably with those calculated using the Georgia Pine Plantation Simulation (GAPPS) model (Burgan et al. 1989). While every point on the diagram cannot be compared with the GAPPS model, for $D_q > 6$ in., yield predictions are typically only 5% greater using the density-management diagram. Differences in site height estimation between systems are usually within 1%. One reason for the difference in predicted yields between systems relates to the minimum tree diameter included in the volume calculations (i.e., 3 in. and 4.5 in. for the density-management diagram and GAPPS model, respectively). This difference in minimum size class causes total volume estimations of the diagram and GAPPS model to diverge with smaller values of D_q.

Table 3. Mean residuals (R) from verification data set for fitted nonlinear regression equations as a percentage of quadratic mean diameter (D_q) and mean stem volume ($\bar{V}$) for unfertilized and fertilized slash pine plantations by CRIFF soil group.[1]

	CRIFF soil group						
	A	B	C	D	E	F	G
Unfertilized							
n	7	73	80	93	44	19	9
$R - D_q$[2]	6.36	0.39	−1.86***	−0.35	0.32	−0.06	1.49
$R - \bar{V}$	4.17	−1.39	0.18	0.96*	0.55	1.10	13.74**
Fertilized							
n	12	125	74	127	42	26	15
$R - D_q$	−2.92*	0.55	−0.76**	−0.17	1.50**	−1.08**	2.22
$R - \bar{V}$	8.78**	−0.56	−0.33	−0.44	−1.40	0.83	5.59***

[1] After Fisher and Garbett (1980).
[2] *$P < 0.1$; **$P < 0.05$; ***$P < 0.01$.

DIAGRAM USE

To fully utilize the diagram, a manager should know how to locate and read values off the diagram, derive stand age using site-index curves, plot normal stand development, and determine the upper and lower limits of stocking.

Locating and Reading Values

Two variables are needed to position a stand on the diagram. The easiest obtained variables to use are N, D_q and site height. When using the principal variables of D_q and N, the stand is positioned by simply locating the intersection of these two variables on the y and x axes, respectively. If site height is used, the stand is located where the height isoline intersects with the appropriate value of either D_q or N (i.e., if site height is not a multiple of 10, height would be estimated by interpolating between the two closest lines). Total standing volume is then determined by interpolating between the volume isolines. For example, if a stand has a D_q of 7 in. and 250 trees/ac, it would have a total standing volume of about 1800 ft^3/ac (Figure 4; the triangle represents the stand location on the diagram).

Determining Stand Age

The age of a plantation can be determined using both the site-height isolines on the diagram and appropriate site-index curves. For example, if site-index = 70 (base age 25 yr), any stand positioned on the diagram's site height 70 ft iso-

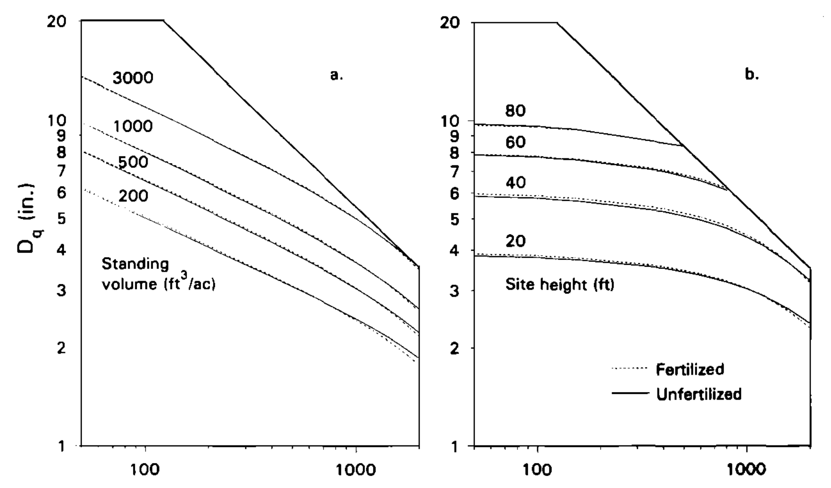

Figure 3. Effects of fertilization on the (a) standing volume and (b) site height isolines of the density-management diagram.

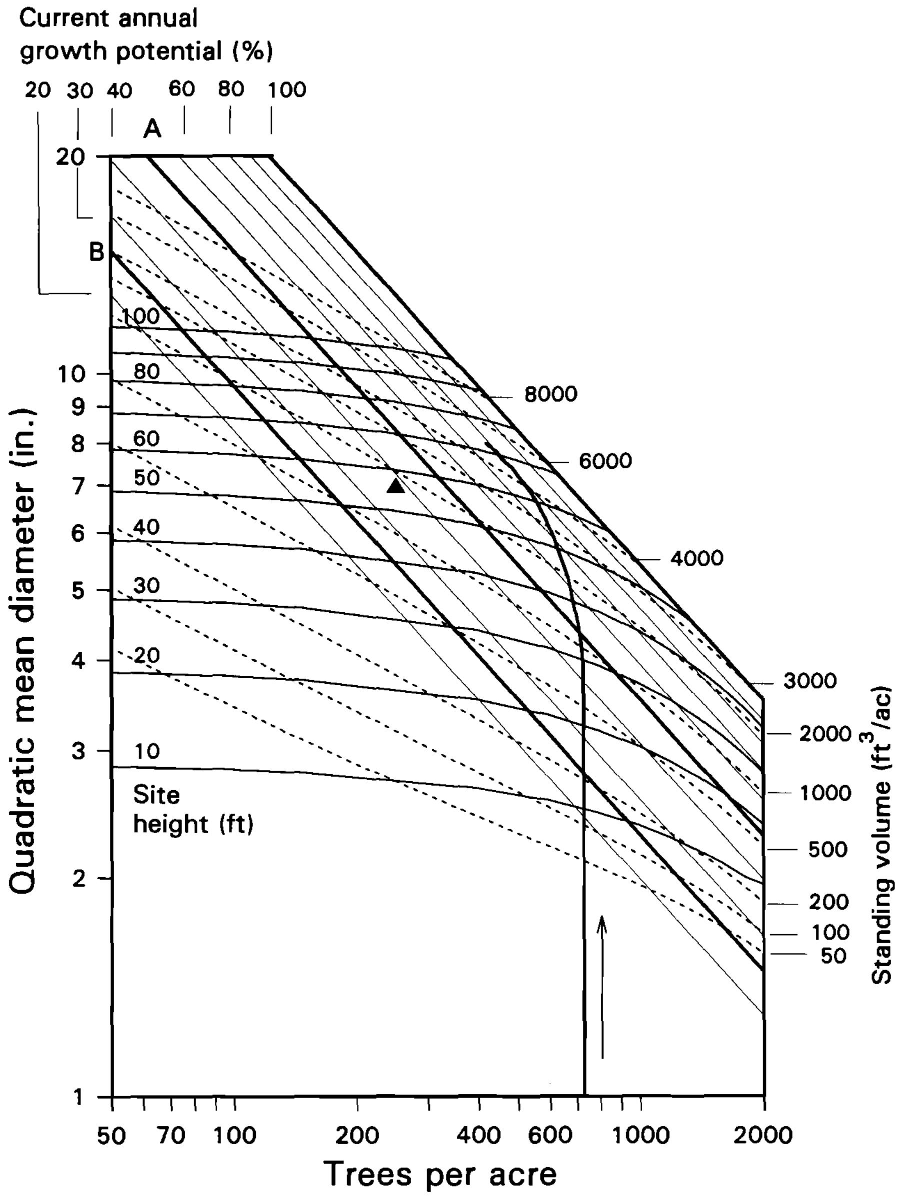

Figure 4. Relation between quadratic mean diameter and trees per acre for an untreated slash pine plantation with an initial planting density of 726 trees/ac (site index 60). Arrow indicates direction of change through time. Curve above 50% current annual growth potential determined using GAPPS simulation model. Also shown is the location of a single stand with 250 trees/ac and quadratic mean diameter of 7 in. (depicted by the triangle). According to the diagram, such a stand would have a volume of 1800 ft³/ac and a site height of 56 ft.

line would be 25 years old. If, on the other hand, site-height is 50 ft in a plantation with a known site-index of 70, the stand would be 15 years old according to the site-index curves of Bailey et al. (1982).[1]

[1] $H = S\{1.0886[1 - \exp(-0.10035 \cdot A)]\}^{2.0669}$ where H = site height (ft), S = site-index (base age 25 yr), and A = plantation age (yr).

Plotting Stand Development

Unlike the Gingrich-type stocking charts, stand development can be plotted on the density-management diagram by following two simple rules. First, when a stand is positioned below the upper stocking limit, D_q will increase without competition-related mortality. Second, when the upper stocking limit is exceeded, self-thinning will cause changes in D_q and N to proceed parallel to the current annual growth potential isolines. The isoline along which a stand will self-thin cannot be predicted *a priori*. Growth and yield simulators show that the average, unmanaged stand self-thins along a 70 to 80% stocking line (data from McCarter and Long 1986 and from GAPPS, Burgan et al. 1989). Therefore, in lieu of other information (e.g., past records of stand growth on a given site), it is reasonable to assume that a self-thinning stand will follow the 75% current annual growth potential line. The actual course of stand development that leads to self-thinning occurs more gradually than these two rules imply, and several attempts have been made to describe it mathematically (Smith and Hann 1984, Tait 1988). Knowing the exact transition is not crucial, however, to use the diagram effectively.

Upper and Lower Stocking Limits

Establishing an upper stocking limit is necessary to avoid competition-related mortality and to maintain individual tree vigor. Since the diagram specifies stocking in terms of growth potential, a manager may be tempted to maximize growth by prescribing the highest stocking levels possible. This decision undertakes certain risks, however. Development of highly stocked stands is unpredictable because of stochastic episodes of mortality and increased susceptibility to insect and disease outbreaks (Drew and Flewelling 1977, Lorio 1980, Long and Smith 1984). Such losses would interfere with, if not eliminate, a forester's ability to meet management objectives that depended on high stocking.

The upper stocking limit is commonly set using two criteria: (1) the onset of competition-related mortality or self-thinning (Drew and Flewelling 1977); and (2) the minimum live-crown ratio necessary for a prompt thinning response: ≈40% (Long 1985). Self-thinning in both fertilized and un-

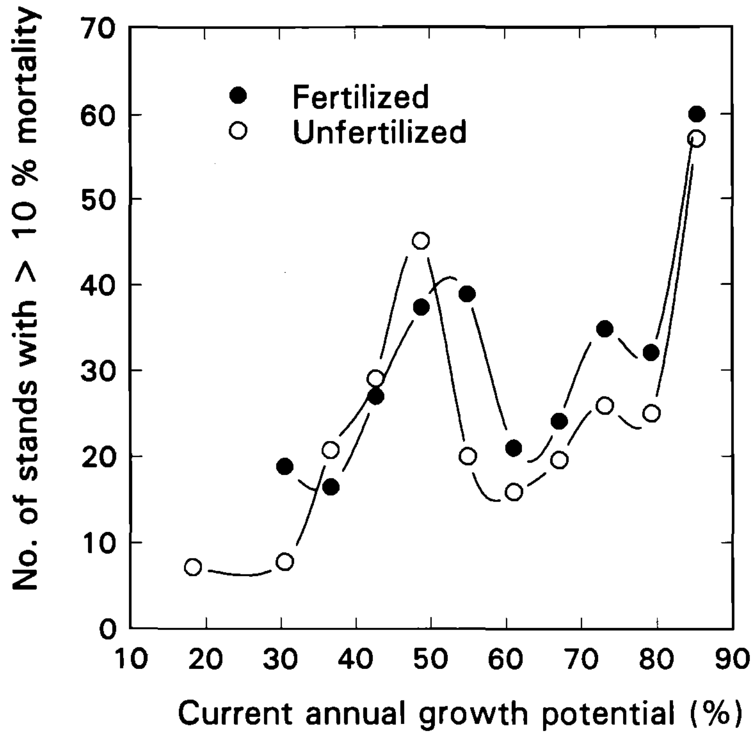

Figure 5. Number of plantations with greater than 10% mortality between measurement periods as a function of current annual growth potential.

fertilized slash pine plantations seems to begin at about 45% current annual growth potential (Figure 5). In addition, a 40% live crown ratio corresponds to about 55% current annual growth potential (data from Bennett 1971). We therefore averaged these two levels and set the upper stocking limit for slash pine to 50% current annual growth potential (see A line, Figure 2). This upper limit corresponds to that reported for coastal Douglas-fir and radiata pine (*Pinus radiata*) (Drew and Flewelling 1977, 1979).

The lower stocking limit corresponds to the lower limit of full-site occupancy that occurs shortly after canopy closure (Strub et al. 1975, Long and Smith 1984). Canopy closure as a function of D_q and N can be estimated from a relationship between *dbh* and crown width (W) for open-grown trees. Using a relationship we developed for slash pine,[2] and assuming triangular spacing, the lower stocking limit was set at 25% current annual growth potential (see B line, Figure 2).

Examples

Use of the density-management diagram will be illustrated by examining three alternative density-management regimes for producing pulpwood-sized materials. These examples are not intended to represent the full range of the diagram's application, however.

The first alternative density-management regime represents a typical prescription for producing pulpwood on a 25-year rotation. In this example, site-index is assumed to be 60, and the initial planting density is 726 trees/ac (6 × 10 ft spacing). The stand is allowed to grow without any intermediate treatments such as thinning or fertilization. The pattern of stand development above the upper stocking limit is plotted using GAPPS model output (Figure 4). This example illustrates the transition from having no competition-related mortality (below 50% current annual growth potential) to increasingly greater amounts of mortality (≈550 trees/ac) as stocking reaches its highest levels (≈80%). At rotation, this prescription yields a final volume of 3800 ft^3/ac and a D_q of 6.7 in. (Table 4).

Figure 6 (panel I) depicts the strategy of a second prescription that allows a stand to grow unthinned to the 50% stocking level at rotation age. According to the diagram, maximum yield would be achieved under these constraints with an initial planting density of 325 trees/ac. At rotation (25 yr), the final harvest volume would be 2400 ft^3/ac (Table 5). The 37% reduction in yield resulting from this density-management regime represents the cost of maintaining good individual tree vigor throughout the rotation and avoiding the attendant risks associated with high stocking levels. Part of the reduction in final harvest volume is offset, however, by a 9% increase in D_q relative to the previous example.

In the third example (Figure 6; panel II), a low thinning is incorporated in the preceding prescription to increase net yield and to provide an early financial return. The initial planting density for this prescription can be determined from the diagram by stairstepping backwards (down) between the upper (50%) and lower (25%) stocking limits, starting from the desired ending point of the rotation. The decision to stop stairstepping and extend the line to the horizontal axis depends on the minimum merchantable size and the economics of precommercial thinning. In this example, we assumed that the minimum merchantable D_q was 4 in. and that precommercial thinning was uneconomical. Therefore, when the line dropped below D_q = 4 in., it was extended to the horizontal axis, giving an initial planting density of about 810 trees/ac.

Thinning from below will increase D_q since the smaller trees are removed. To simulate this effect on the diagram, the horizontal legs of the "steps" assume the

[2] $W = 2.804 + 2.201 \cdot dbh$; ($dbh < 20$ in., $n = 28$, $r^2 = 0.89$, $s_{y \cdot x} = 2.93$ ft)

Table 4. Planting and harvest data predicted for a conventional pulpwood regime using the GAPPS model (Burgan et al. 1989).

Conventional pulpwood regime Operation	Age	H_s	Trees/ac	D_q	Yield
Planting			726		
Final harvest	25	61(+1)[1]	550	6.7	3800(+200)
MAI = 152 ft^3 ac^{-1} yr^{-1}					

[1] Values in parentheses denote deviation in predictions between the density-management diagram and GAPPS model.

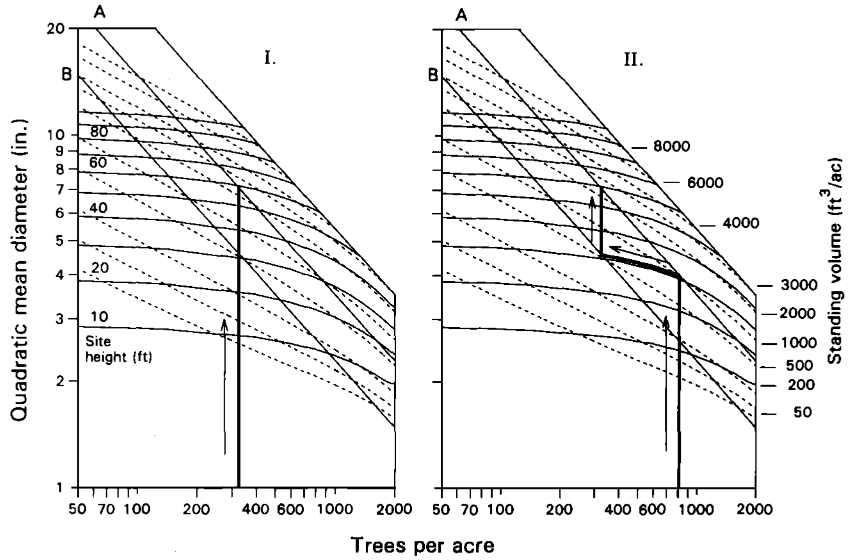

Figure 6. Alternative density-management regimes for slash pine plantations managed for a pulpwood objective: no thinning (I); and a single low thinning (II). Arrows show the direction of change through time.

shape of the nearest site height isoline. Since these isolines increase from right to left, the value of D_q increases with thinning. This technique simultaneously maintains a constant site height, which is also expected with a low thinning.

The amount of wood removed by thinning is the difference in volume before and after treatment. For this example, thinning removed about 400 ft^3/ac at age 12 years and increased the mean annual increment by 13% compared to example two (Table 5).

The density-management diagram shows the interrelationships among stand variables at equilibrium conditions. Consequently, it cannot accurately describe stand conditions immediately after thinning (i.e., differences in crown structure between a thinned and unthinned stand at the same combination of D_q and N). The technique of simulating thinning by moving from the upper to lower stocking limits, parallel with the nearest height isoline, is an attempt to compensate for this limitation. The effect of thinning on the diagram's accuracy can be minimized by maintaining a stand within the suggested upper and lower stocking limits. Heavy thinnings in highly stocked stands, for example, would probably preclude the accurate use of the diagram; additional validation testing is warranted for thinned stands because of these inherent limitations.

Allowing for Midrotation Fertilization

Fertilization is taken into account in the diagram by determining its effect on changes in the height growth of site trees and, hence, site quality. Treatment-specific height growth equations can be used for this purpose and have been developed for slash pine stands receiving midrotation fertilizer applications (Bailey et al. 1989). In essence, fertilization reduces the time required for a stand to achieve a given site height. For example, fertilizing a stand with 726 trees/ac at 10 years of age (site-index 60) reduces by 3 years the length of time required to attain a site height of 60 ft. At age 25 years, the site height of the fertilized stand would be 4 ft taller than the unfertilized stand which, according to the diagram, would increase the final harvest volume by about 400 ft^3/ac and the mean annual increment by 11%.

SUMMARY

The density-management diagram for lower Coastal Plain slash

Table 5. Mensurational data predicted from the density-management diagram for two alternative pulpwood-management regimes for slash pine plantations (site index = 60). D_q = quadratic mean diameter (in.), H_s = site height (ft), and age and yield expressed in years and ft^3/ac, respectively. Mean annual increment (MAI) is total yield divided by plantation age at final harvest.

I. Alternative pulpwood regime Operation	Age	H_s	Trees/ac	D_q	Yield
Planting			325		
Final Harvest	25	60	325	7.3	2400
MAI = 96 ft^3 ac^{-1} yr^{-1}					

II. Alternative pulpwood regime with a low thinning			Trees/ac		D_q		
Operation	Age	H_s	Before	After	Before	After	Yield
Planting			810				
Thinning	12	31	810	325	3.9	4.5	400
Final harvest	25	60	325		7.3		2400
Total yield							2800
MAI = 112 ft^3 ac^{-1} yr^{-1}							

pine plantations (Figure 2) provides resource managers with a versatile visual tool to aid decision making. Using standard mensurational data (e.g., D_q and N), a manager can determine total standing volume, average height of dominant and codominant trees, and relative growth potential. The diagram can also be used to determine the probable direction of future stand development, and its utility is greatly enhanced since it applies equally well to fertilized and unfertilized plantations.

With little training and the appropriate site-index curves, the diagram can be used by managers to select and compare alternative density-management regimes for meeting specific management objectives. The tradeoff in maximizing individual tree growth versus stand level performance is also easily illustrated using the diagram. In addition, if increases in height growth due to fertilization can be determined, volume responses can be estimated.

As with any statistical model, predictions should not be extrapolated outside the range of the original data. Therefore, the diagram presented for slash pine has greatest utility in developing and comparing alternative density-management strategies for meeting either pulpwood or chip-n-saw objectives on CRIFF soil groups B–F. □

Literature Cited

Arney, J.D. 1985. A modeling strategy for the growth projection of managed stands. Can. J. For. Res. 15:511–518.

Bailey, R.L., T.M. Burgan, and E.J. Jokela. 1989. Fertilized midrotation-aged slash pine plantations—stand structure and yield prediction models. South. J. Appl. For. 13:76–80.

Bailey, R.L., L.V. Pienaar, B.D. Shiver, and J.W. Rheney. 1982. Stand structure and yield of site-prepared slash pine plantations. Georgia Agric. Exp. Stn. Res. Bull. No. 291. 83 p.

Bennett, F.A. 1971. The role of thinning and some other problems in management of slash pine plantations. USDA For. Serv. Res. Pap. SE-86. 14 p.

Burgan, T.M., R.L. Bailey, and J.R. Brooks. 1989. GAPPS (GeorgiA Pine Plantation Simulator) an interactive microcomputer program for stand structure, yield, growth, and financial analysis of thinned, site-prepared slash and loblolly pine plantations (Version 2.0988). Sch. For. Resour., Univ. of Ga, Athens. 36 p.

Clutter, J.L., and D.M. Belcher. 1978. Yield of site-prepared slash pine plantations in the lower coastal plain of Georgia and Florida. P. 53–70 *in* Growth models for long-term forecasting of timber yields. Proc. IUFRO, Publ. FWS-1-78, Sch. For. Wildl. Resour., Virginia Polytech. Inst., State Univ., Blacksburg.

Colbert, S.R., E.J. Jokela, and D.G. Neary. 1990. Effects of annual fertilization and sustained weed control on dry matter partitioning, leaf area, and growth efficiency of juvenile loblolly and slash pine. For. Sci. 36:995–1014.

Dell, T.R., et al. 1979. Yields of unthinned slash pine plantations on cutover sites in the west Gulf Region. USDA For. Serv. Res. Pap. SO-134. 5 p.

Drew, T.J., and J.W. Flewelling. 1977. Some recent Japanese theories of yield-density relationships and their application to Monterey pine plantations. For. Sci. 22:517–534.

Drew, T.J., and J.W. Flewelling. 1979. Stand density management: An alternative approach and its application to Douglas-fir plantations. For. Sci. 25:518–532.

Fisher, R.F., and W.S. Garbett. 1980. Response of semimature slash and loblolly pine plantations to fertilization with nitrogen and phosphorus. Soil Sci. Soc. Am. J. 44:850–854.

Flewelling, J.W. 1981. A comparison of three stand density indices for predicting gross volume in plantations of loblolly pine. Diss., Univ. of Georgia, Athens. 83 p.

Gingrich, S.F. 1967. Measuring and evaluating stocking and stand density in upland hardwood forests in the central states. For. Sci. 7:35–42.

Jokela, E.J., R.B. Harding, and C.A. Nowak. 1989. Long-term effects of fertilization on stem form, growth relations, and yield estimates of slash pine. For. Sci. 35:832–842.

Kidder, G., N.B. Comerford, and A.V. Mollitor. 1987. Fertilization of slash pine plantations. Circ. No. 735. Florida Coop. Ext. Serv., Inst. of Food and Agric. Sci., University of Florida, Gainesville. 5 p.

Long, J.N. 1985. A practical approach to density management. For. Chron. 61:23–27.

Long, J.N., and F.W. Smith. 1984. Relation between size and density in developing stands: A description and possible mechanism. For. Ecol. Manage. 7:191–206.

Lorio, P.L. 1980. Loblolly pine stocking levels affect potential for southern pine beetle infestation. South. J. Appl. For. 4: 162–165.

McCarter, J.B., and J.N. Long. 1986. A lodgepole pine density management diagram. West. J. Appl. For. 1:6–11.

Reineke, L.H. 1933. Perfecting a stand density index for even-aged forests. J. Agric. Res. 46:627–638.

Rogers, R. 1983. Guides for thinning shortleaf pine. P. 217–225 *in* Proc. 2nd Bienn. South. Silvic. Conf., Jones, E.P., Jr. (ed.). USDA For. Serv. Gen. Tech. Rep. SE-24.

Schroeder, H.T., W.L. Hafley, and F.A. Bennett. 1979. Yield prediction for unthinned natural slash pine stands. For. Sci. 25:25–30.

Smith, N.J. 1989. A stand-density control diagram for western red cedar, *Thuja plicata*. For. Ecol. Manage. 27:235–244.

Smith, N.J., and D.W. Hann. 1984. A new analytical model based on the $-3/2$ power rule of self-thinning. Can. J. For. Res. 14: 605–609.

Strub, M.R., R.B. Vasey, and H.E. Burkhart. 1975. Comparison of diameter growth and crown competition factor in loblolly pine plantations. For. Sci. 21: 427–431.

Tait, D.E. 1988. The dynamics of stand development: A general stand model applied to Douglas-fir. Can. J. For. Res. 18: 696–702.