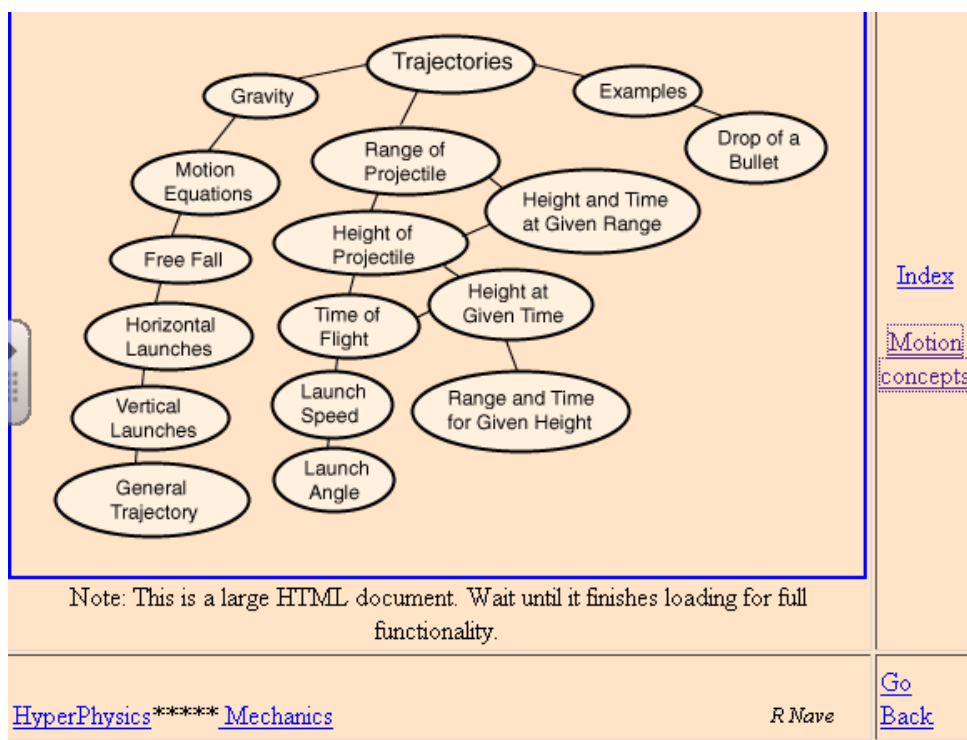
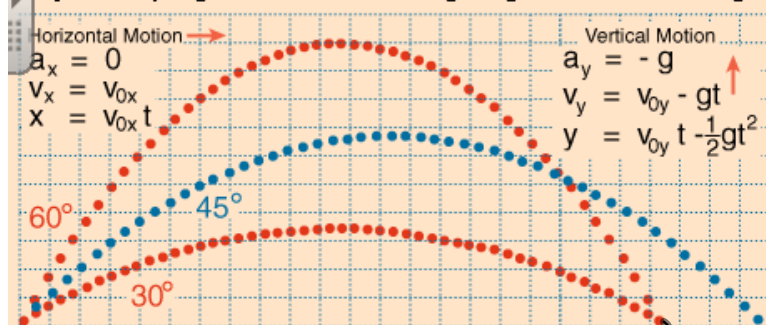


Projectile Launched at an angle



General Ballistic Trajectory

The motion of an object under the influence of gravity is determined completely by the acceleration of gravity, its launch speed, and launch angle provided air friction is negligible. The horizontal and vertical motions may be separated and described by the general [motion equations](#) for constant acceleration. The initial vector components of the velocity are used in the equations. The diagram shows trajectories with the same launch speed but different launch angles. Note that the 60 and 30 degree trajectories have the same range, as do any pair of launches at complementary angles. The launch at 45 degrees gives the maximum range.


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[Mechanics](#)

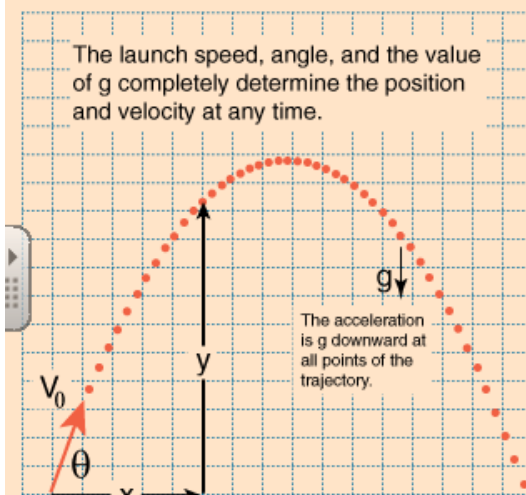
Range = Δx

R
Nave

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45° Launch gives longest range

The launch speed, angle, and the value of g completely determine the position and velocity at any time.



For launch velocity $v_0 =$ m/s, launch angle $\theta =$ degrees:

At time $t =$ sec:

Horizontal Motion →

$a_x = 0$
 $v_x = v_{0x}$

Horizontal velocity
 $v_x =$ m/s.
 $x = v_{0x} t$

Horizontal distance
 $x =$ m.

Vertical Motion ↑ +
 Upward chosen as positive direction for y motion.
 $a_y = -g = -9.8 \text{ m/s}^2$
 $v_y = v_{0y} - gt$

Vertical velocity
 $v_y =$ m/s.
 $y = v_{0y} t - \frac{1}{2} gt^2$

Vertical position
 $y =$ m.

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Range of Trajectory

The acceleration is g downward at all points of the trajectory.

Range = $R = \frac{v_0^2 \sin 2\theta}{g}$

$v_{ix} = -v_{fx}$

The basic **motion equation**
 $x = v_{0x} t$
 can be used to find the range.
 By symmetry, the total **time of flight** is equal to twice the time at the peak:
 $t_{\text{range}} = 2t_{\text{peak}} = \frac{2v_{0y}}{g}$
 This gives:
 $R = \frac{2v_{0x} v_{0y}}{g}$
 $R = \frac{2v_0^2 \sin \theta \cos \theta}{g}$
 $R = \frac{v_0^2 \sin 2\theta}{g}$ Calculation
 using the **trig identity**:
 $\sin 2\theta = 2 \sin \theta \cos \theta$.

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$$\Delta x = \frac{v_i^2 \sin 2\theta}{g} (\sin \theta \cos \theta)$$

Height of Trajectory

$h = \frac{v_{0y}^2}{2g} = \frac{v_0^2 \sin^2 \theta}{2g}$

The basic **motion equation**
 $y = \bar{v}_y t$
 can be used to find the height.
 The average vertical speed is:

$$\bar{v}_y = \frac{v_{0y} + 0}{2} = \frac{v_{0y}}{2}$$

The time at the peak is obtained by solving for the time at zero vertical speed:

$$0 = v_{0y} - gt_{\text{peak}}$$

This gives:

$$t_{\text{peak}} = \frac{v_{0y}}{g}$$

and substituting:

$$h = y_{\text{peak}} = \frac{v_{0y}}{2g}$$

Calculator

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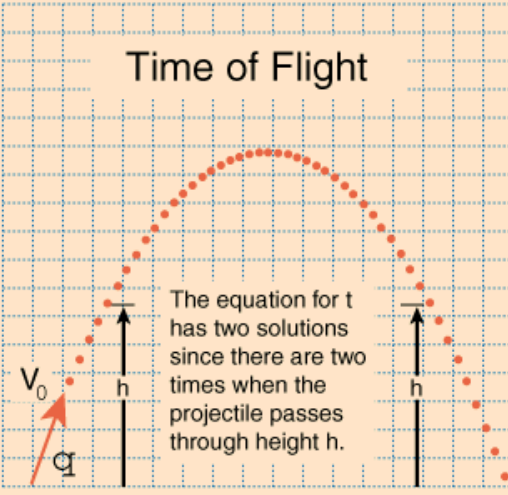
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R Nave

$y_{\text{max}} = \frac{(v_{iy})^2}{2g}$

 $\Delta y = \frac{1}{2}(v_{iy} + v_{fy})t$

Time of Flight



The equation for t has two solutions since there are two times when the projectile passes through height h .

The basic **motion equation**

$$h = v_{0y} t - \frac{1}{2} g t^2$$

can be used to find the time of flight at height h , giving:*

$$t = \frac{v_{0y}}{g} \pm \sqrt{\frac{v_{0y}^2}{g^2} - \frac{2h}{g}}$$

Note that there is no real solution if

$$\frac{2h}{g} > \frac{v_{0y}^2}{g^2} \text{ or } h > \frac{v_{0y}^2}{2g}$$

since such values of h are above the peak of the trajectory. For the value $h=0$:

$$t = 0 \text{ and } t = \frac{2v_{0y}}{g}$$

Calculation

*quadratic formula

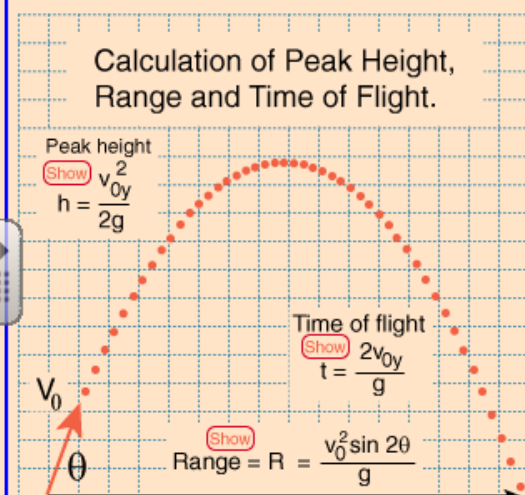
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R Nave

$$\Delta t = \frac{2 v_{i,y}}{g}$$

<p>Calculation of Peak Height, Range and Time of Flight.</p>  <p>Peak height Show $h = \frac{v_{0y}^2}{2g}$</p> <p>Time of flight Show $t = \frac{2v_{0y}}{g}$</p> <p>Show Range = $R = \frac{v_0^2 \sin 2\theta}{g}$</p>	<p>For launch velocity $v_0 =$ <input type="text"/> m/s, launch angle $\theta =$ <input type="text"/> degrees, The horizontal range is $R =$ <input type="text"/> m. The total time of flight is $t =$ <input type="text"/> s. The peak height is $h =$ <input type="text"/> m.</p>	<p>Index</p> <p>Trajectory concepts</p>
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R
Nave

Will it clear an object?



Where will it land?

Ballistic

Launch Velocity

Angle of Launch

Pg 110 #34

1700 $\frac{m}{s}$

55°

y	x
$\Delta y =$	$\Delta x = ?$
$v_{iy} =$	$v_x =$
$v_{fy} =$	
$a = -9.8 \frac{m}{s^2}$	

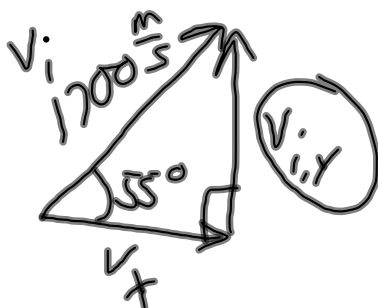
 $\Delta x = ?$
 $\Delta t = ?$
 $\sin \theta$ (cos θ)

$$\Delta x = \frac{2 v_i^2 \sin 2\theta}{9.8 \frac{m}{s^2}} = \frac{2 (1700 \frac{m}{s})^2 \sin 2(55^\circ)}{9.8 \frac{m}{s^2}}$$

$$= 277000 \text{ m}$$

$$(\sin 55^\circ)(\cos 55^\circ) = \sin 2(55^\circ)$$

$$\Delta t = \frac{2 v_{iy}}{9.8 \frac{m}{s^2}} = \frac{2 (1393 \frac{m}{s})}{9.8 \frac{m}{s^2}} = 284 \frac{s}{5}$$

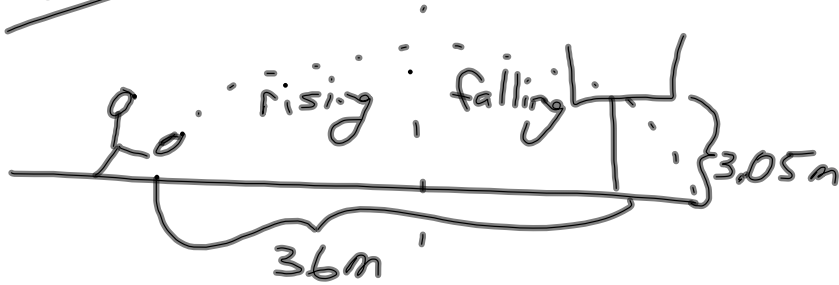


$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$v_{iy} = (\sin 55^\circ) 1700 \frac{m}{s}$$

$$= 1393 \frac{m}{s}$$

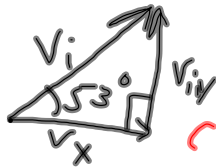
Pg 110 #35



$$v_i = 20 \frac{m}{s}$$

$\theta = 53^\circ$ to horizontal

a) is the ball above or below the crossbar?
 $\therefore \Delta y @ 36m$



$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$v_x = v_i \cos 53^\circ$$

$$= 20 \frac{m}{s} (\cos 53^\circ) = 12 \frac{m}{s}$$

$$v_{iy} = v_i \sin 53^\circ = 16 \frac{m}{s}$$

y	x
$\Delta y = ?$	$\Delta x = 36m$
$v_{iy} = 16 \frac{m}{s}$	$v_x = 12 \frac{m}{s}$
$v_{fy} = \text{X}$	
$a = -9.8 \frac{m}{s^2}$	

$$\Delta t = 3s$$

$$v_x = \frac{\Delta x}{\Delta t} \therefore \Delta t = \frac{\Delta x}{v_x} = \frac{36m}{12 \frac{m}{s}} = 3s$$

$$\Delta y = \frac{1}{2} a t^2 + v_{iy} t$$

$$\frac{1}{2} (-9.8 \frac{m}{s^2}) (3s)^2 + (16 \frac{m}{s}) (3s) = 3.9m$$

b) is it rising or falling? \therefore clear goal by 0.85m

$$y_{max} = \frac{v_{iy}^2}{2(9.8 \frac{m}{s^2})}$$

falling