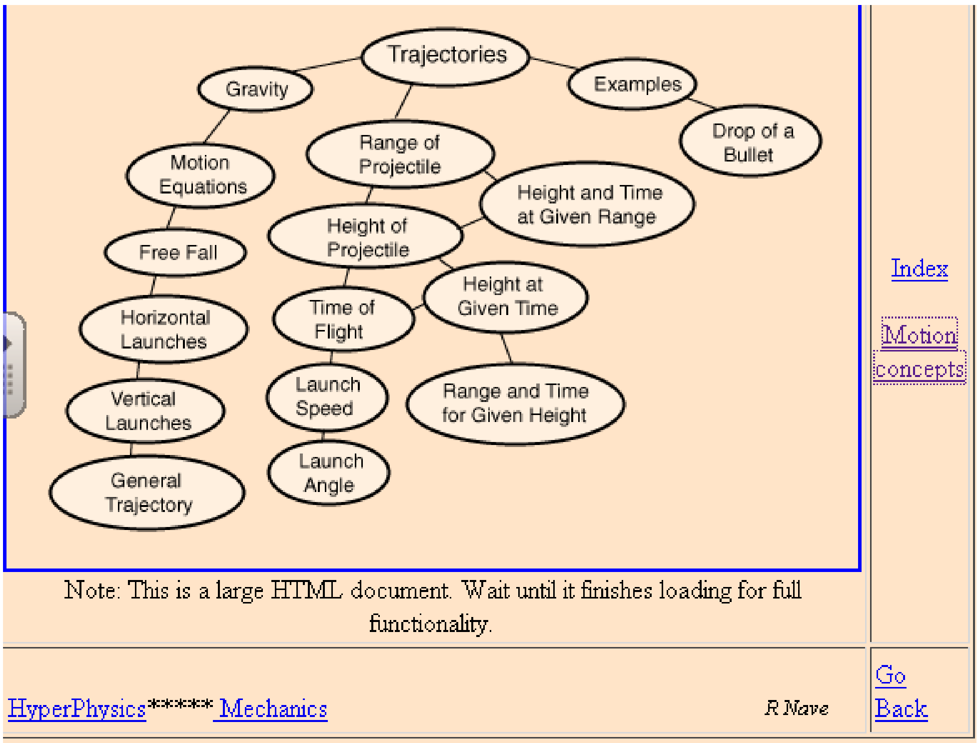
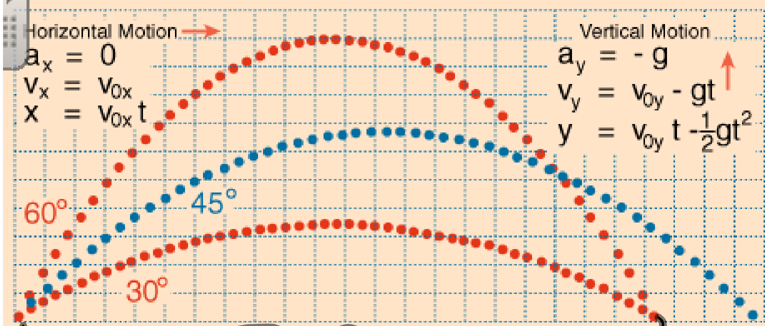
Projectile Launched at an angle



General Ballistic Trajectory

The motion of an object under the influence of gravity is determined completely by the acceleration of gravity, its launch speed, and launch angle provided air friction is negligible. The horizontal and vertical motions may be separated and described by the general motion equations for constant acceleration. The initial vector components of the velocity are used in the equations. The diagram shows trajectories with the same launch speed but different launch angles. Note that the 60 and 30 degree trajectories have the same range, as do any pair of launches at complementary angles. The launch at 45 degrees gives the maximum range.



Range = Δx

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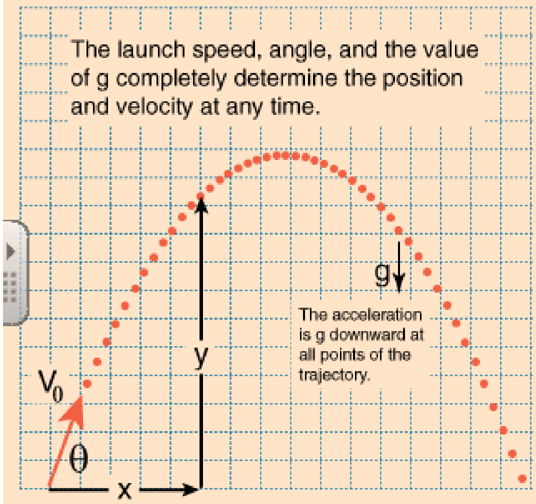
45° Lauch gives longest range

At time t = [ ] sec:



For launch velocity v_0 = [ ] m/s, launch angle θ = [ ] degrees:

Horizontal Motion

$a_x = 0$

$v_x = v_{0x}$

Horizontal velocity

v_x = [ ] m/s.

$x = v_{0x}t$

Horizontal distance

x = [ ] m.

Vertical Motion +

Upward chosen as positive direction for y motion.

$a_y = -g = -9.8\text{m/s}^2$

$v_y = v_{0y} - gt$

Vertical velocity

v_y = [ ] m/s.

$y = v_{0y}t - \frac{1}{2}gt^2$

Vertical position

y = [ ] m.

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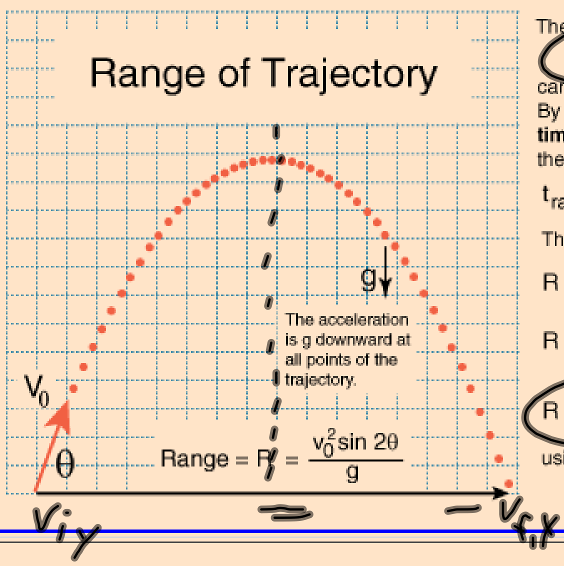

The basic motion equation

$$x = v_{0x}\, t$$

can be used to find the range. By symmetry, the total **time of flight** is equal to twice the time at the peak:

$$t_{range} = 2t_{peak} = \frac{2v_{0y}}{g}$$

This gives:

$$R = \frac{2v_{0x}\, v_{0y}}{g}$$

$$R = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

Calculation

using the **trig identity**:

$\sin 2\theta = 2\sin\theta \cos\theta.$

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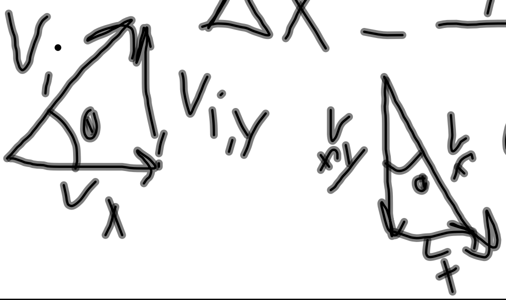
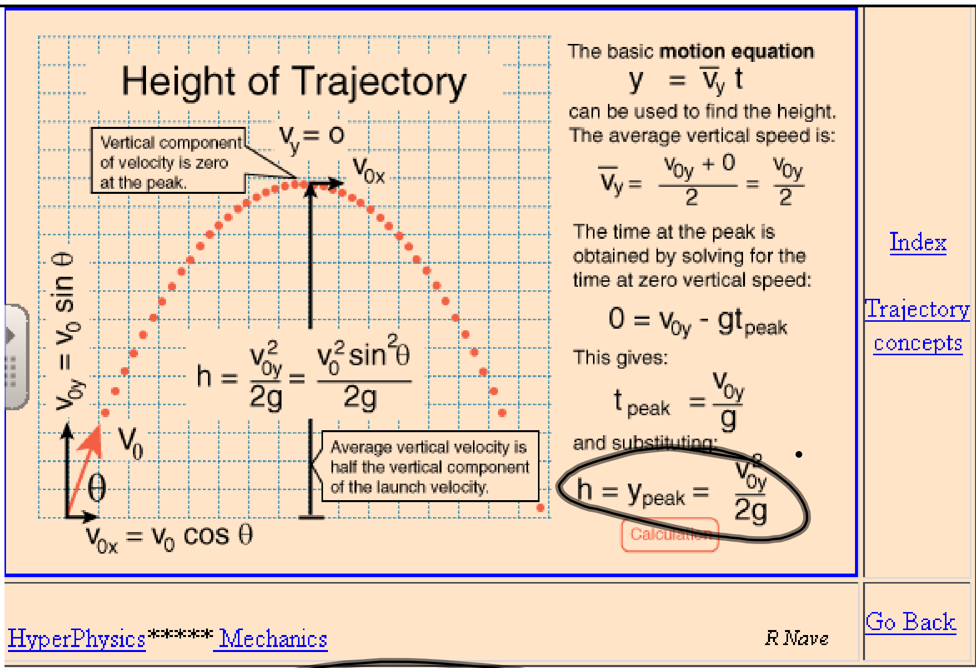
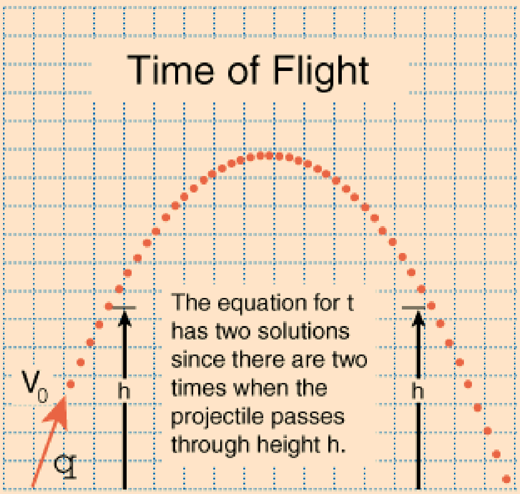
$$\Delta x = \frac{v_i^2 \sin 2\theta}{g} \quad (\sin\theta \cos\theta)$$





$$y_{max} = \frac{(v_{i,y})^2}{2g}$$

$$\Delta y = \frac{1}{2}(v_{i,y} + \cancel{v_{f,y}})t$$



The basic **motion equation**

$$h = v_{0y} t - \frac{1}{2} g t^2$$

can be used to find the time of flight at height h, giving:*

$$t = \frac{v_{0y}}{g} \pm \sqrt{\frac{v_{0y}^2}{g^2} - \frac{2h}{g}}$$

Note that there is no real solution if

$$\frac{2h}{g} > \frac{v_{0y}^2}{g^2} \quad \text{or } h > \frac{v_{0y}^2}{2g}$$

since such values of h are above the peak of the trajectory. For the value h=0:

$$t = 0 \text{ and } t = \frac{2v_{0y}}{g}$$

Calculation

*quadratic formula

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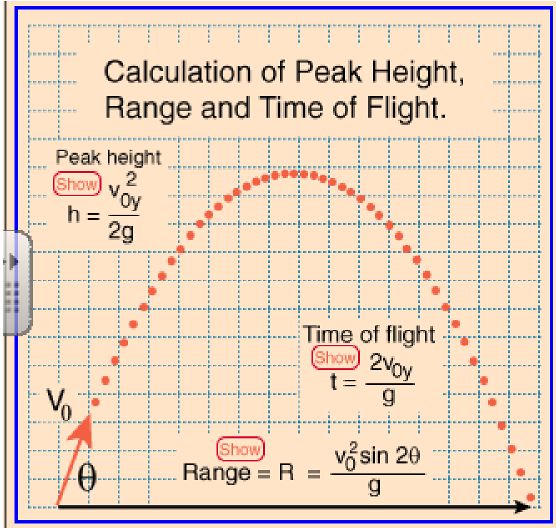
$$\Delta t = \frac{2 v_{i,y}}{g}$$



For launch velocity
v_0 = ______ m/s,
launch angle
θ = ______ degrees,
The horizontal range is
R = ______ m.
The total time of flight is
t = ______ s.
The peak height is
h = ______ m.

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Will it clear an object? ✓

Where will it land? Ballistic

Launch Velocity

Angle of Launch

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y	x
$\Delta y =$	$\Delta x = ?$
$v_{iy} =$	$v_x =$
$v_{fy} =$	
$a = -9.8\,\frac{m}{s^2}$	

$\Delta x = ?$

$\Delta t = ?$

$\sin\theta(\cos\theta)$

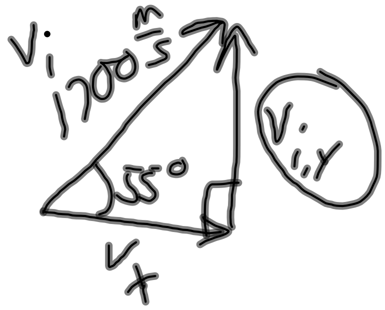
$$\Delta x = \frac{2 v_i^2 \sin 2\theta}{9.8\,\frac{m}{s^2}} = \frac{2(1700\,\frac{m}{s})^2 \sin 2(55°)}{9.8\,\frac{m}{s^2}}$$

$$= 277000\ m$$

$$(\sin(55°))(\cos 55°) = \sin 2(55°)$$

$$\Delta t = \frac{2 v_{iy}}{9.8\,\frac{m}{s^2}} = \frac{2(1393\,\cancel{\frac{m}{s}})}{9.8\,\cancel{\frac{m}{s^2}}} = 284\,s$$

$$\sin\theta = \frac{opp}{hyp}$$



$$v_{iy} = (\sin 55°)\,1700\,\frac{m}{s}$$

$$= 1393\,\frac{m}{s}$$

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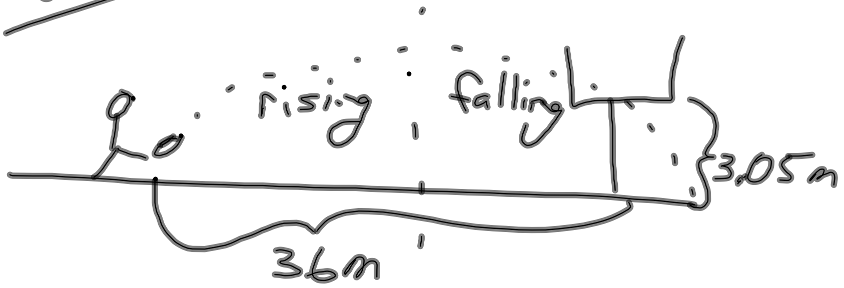
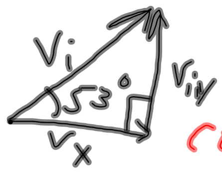


$v_i = 20\frac{m}{s}$

$\theta = 53°$ to horizontal

a) is the ball above or below the crossbar?

$\therefore \Delta y$ @ 36m



$\cos\theta = \frac{adj}{hyp}$

y	x
Δy: ?	$\Delta x = 36m$
$v_{iy} = {+16}\frac{m}{s}$	$v_x = {+12}\frac{m}{s}$
v_{fy} = X	
$a = -9.8\frac{m}{s^2}$	

$v_x = v_i \cos 53°$

$= 20\frac{m}{s}(\cos 53°) = 12\frac{m}{s}$

$v_{iy} = v_i \sin 53° = 16\frac{m}{s}$

$\Delta t = 3s$

$v_x = \frac{\Delta x}{\Delta t} \quad \therefore \quad \Delta t = \frac{\Delta x}{v_x} = \frac{+36m}{+12\frac{m}{s}} = 3s$

$\Delta y = \frac{1}{2}at^2 + v_{iy}t$

$\frac{1}{2}(-9.8\frac{m}{s^2})(3s)^2 + (+16\frac{m}{s})(3s) = 3.9m$

$\therefore$ clear goal by 0.85m

b) is it rising or falling?

$y_{max} = \frac{v_{iy}^2}{\cancel{2(9.8\frac{m}{s^2})}}$

falling