Systems Management

Control Concepts

"It is the mark of a truly intelligent person to be moved by statistics."
George Bernard Shaw

Agenda

- Lecture chapter 11:
  - Control concepts and methods
- Statistical Quality Control
- Student presentations

Assignment

- Read chapter 11
- Coordinate topics and schedule for presentations

Fig. 2.1 The product life cycle

ACQUISITION			UTILIZATION
Conceptual / Preliminary Design	**Detail Design and Development**	**Production / Construction**	**Product use, support, phase-out, and disposal**

Quality

- Contrast popular versus technical meanings of quality, defect
- "Quality is defined as performance to the standard expected by the customer."
  - Fred Smith, CEO, Federal Express
- Quality is "the characteristic or group of characteristics which distinguish one article from another"
  - Walter Shewhart, AT&T Bell Labs, 1931

Quality

- Two aspects of quality: design and conformance
  - Design: What are you trying to build?
  - Conformance: Are you building what you designed?
- Our focus in this chapter: quality of conformance through Statistical Quality Control (SQC)
- Control charts: a tool to determine whether a process is in statistical control

R. A. Fisher

- Conducted agricultural research in Britain
- Developed powerful methods of experimental design

Sir Ronald A. Fisher
1890 - 1962

W. A. Shewhart

- Physicist at AT&T Bell Labs, Post WWI
- Applied statistical techniques to quality control
- Control charts sometimes called "Shewhart Charts"

Walter A. Shewhart
1891 - 1967

W. Edwards Deming

- Statistical sampling methods at Census Bureau, 1940
- Reconstruction of Japan, 1950
  – Three photos at Toyota HQ
- Famous for 14 points
  – Cease dependence on mass inspection (#3)
  – Improve every process (#5)

W. Edwards Deming
1900 - 1993

George Box

- Born in Britain
- Distinguished career at the University of Wisconsin
- Box, Hunter, and Hunter, *Statistics for Experimenters*
- Dr. John MacGregor one of Box’s Ph.D. students

George E. P. Box
1919 -

Genichi Taguchi

- Leader in Japanese quality movement
- "Lack of quality costs society"
- "within tolerance" versus "on target"

Genichi Taguchi
1924 -

Deming's 14 points

1. Create constancy of purpose toward improvement of product and service, with the aim to become competitive and stay in business, and to provide jobs.
2. Adopt the new philosophy. We are in a new economic age. Western management must awaken to the challenge, must learn their responsibilities, and take on leadership for change.
3. **Cease dependence on inspection to achieve quality.** Eliminate the need for inspection on a mass basis by building quality into the product in the first place.

Deming, W. E. (1986) *Out of the Crisis*, MIT Press

Deming’s 14 points

4. End the practice of awarding business on the basis of price tag. Instead, minimize total cost. **Move towards a single supplier for any one item,** on a long-term relationship of loyalty and trust.
5. Improve constantly and forever the system of production and service, to improve quality and productivity, and thus constantly decrease cost.
6. Institute training on the job.

Deming’s 14 points

7. Institute leadership. The aim of supervision should be to help people and machines and gadgets to do a better job. Supervision of management is in need of overhaul, as well as supervision of production workers.
8. Drive out fear, so that everyone may work effectively for the company.
9. **Break down barriers between departments**. People in research, design, sales, and production must work as a team, to foresee problems of production and in use that may be encountered with the product or service.

Deming’s 14 points

10. Eliminate slogans, exhortations, and targets for the work force asking for zero defects and new levels of productivity. Such exhortations only create adversarial relationships, as the bulk of the causes of low quality and low productivity belong to the system and thus lie beyond the power of the work force.

11. Eliminate work standards (quotas) on the factory floor. Eliminate management by objective. Eliminate management by numbers, numerical goals. Substitute leadership.

Deming’s 14 points

12. Remove barriers that rob the hourly worker of his right to pride of workmanship. The responsibility of supervisors must be changed from sheer numbers to quality. Remove barriers that rob people in management and in engineering of their right to pride of workmanship. This means, *inter alia*, **abolishment of the annual or merit rating** and of management by objective.
13. Institute a vigorous program of education and self-improvement.
14. Put everybody in the company to work to accomplish the transformation. The transformation is everybody's work.

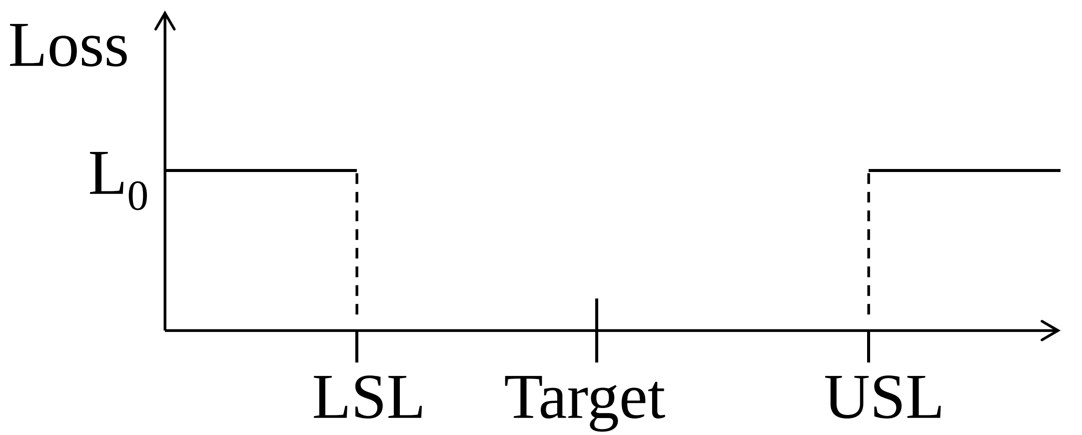
Genichi Taguchi

- Leader in Japanese quality movement
- "Lack of quality costs society"
- "within tolerance" versus "on target"

Genichi Taguchi
1924 -

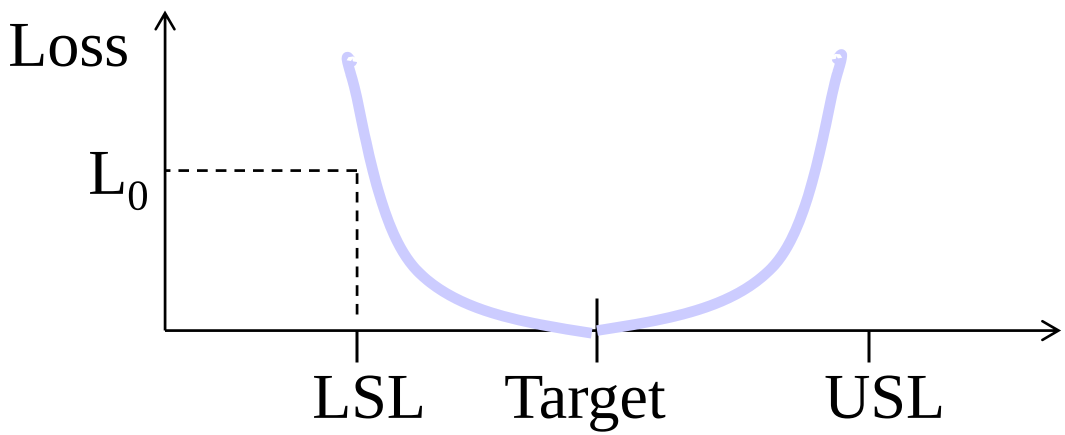
Traditional Interpretation of Specification Limits

- Any product within limits is O.K.
- Any product outside limits is unacceptable



Taguchi's Main Idea

- There are costs associated with any deviation from the target
- Loss function is quadratic: $L = k\,(x-\tau)^2$



Expected Loss

$E(L) = \int k\,(x - \tau)^2 f(x)\,dx$ (definition of expected loss)

$E(L) = k \int [\,(x - \mu) + (\mu - \tau)]^2 f(x)\,dx$ (add and subtract μ)

$E(L) = k \int [(x-\mu)^2 + 2(x-\mu)(\mu-\tau) + (\mu-\tau)^2]\, f(x)dx$

(square term in [ ])

$E(L) = k\, [\int(x-\mu)^2 f(x)dx + 2(\mu-\tau) \int(x-\mu)f(x)dx + (\mu-\tau)^2 \int f(x)dx]$

(Integral of sum is sum of integrals)

Expected Loss

$$E(L) = \int k\,(x - \tau)^2 f(x)\,dx$$

$$E(L) = k \int [\,(x - \mu) + (\mu - \tau)]^2 f(x)\,dx$$

$$E(L) = k \int [(x-\mu)^2 + 2(x-\mu)(\mu-\tau) + (\mu-\tau)^2]\, f(x)dx$$

σ^2 0 1

$$E(L) = k\,[\int(x-\mu)^2 f(x)dx + 2(\mu-\tau) \int(x-\mu)f(x)dx + (\mu-\tau)^2 \int f(x)dx]$$

$$E(L) = k\,[\,\sigma^2 + (\mu - \tau)^2\,]$$

Expected loss includes both bias and variability

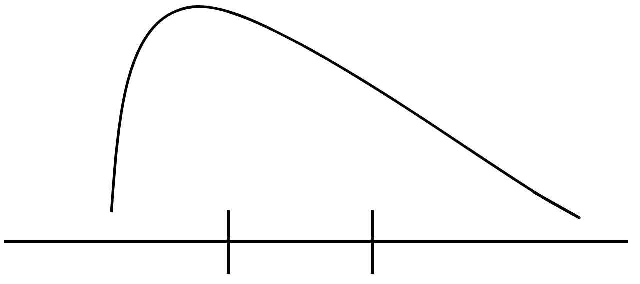
Process Capability Index, C_p

- Assume process is centered; i.e., no bias
- Assume process tolerance is $\mu \pm 3\sigma$
- Then Cp is ratio of specification interval to process variability

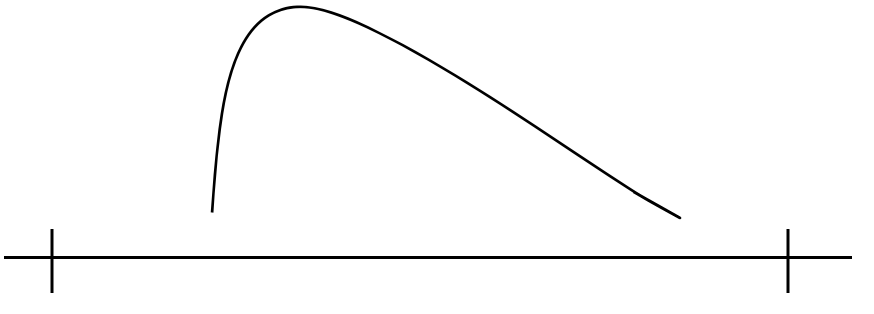
$$C_p = \frac{USL - LSL}{6\sigma}$$

C_p: Three Cases

- $C_p < 1$: BAD



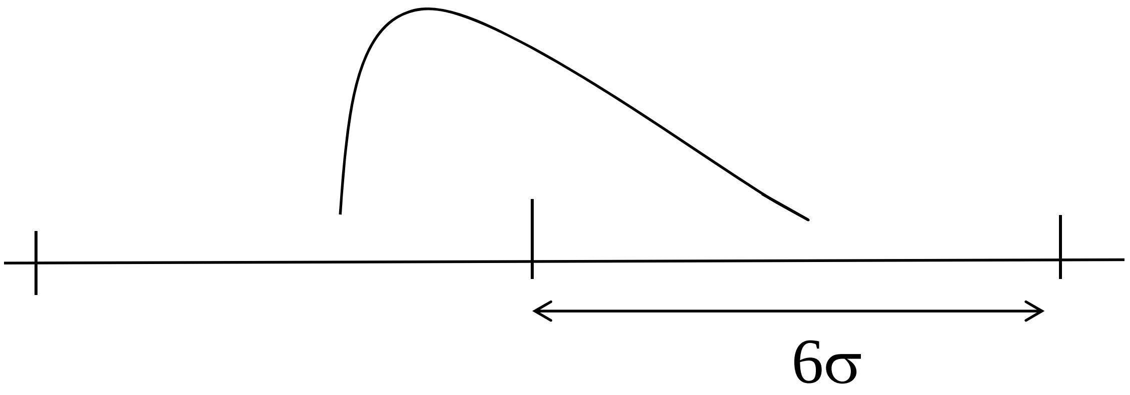
- $C_p = 1$: Marginal
- $C_p > 1$: GOOD



$$C_p = \frac{USL - LSL}{6\sigma}$$

Motorola 6-Sigma Program

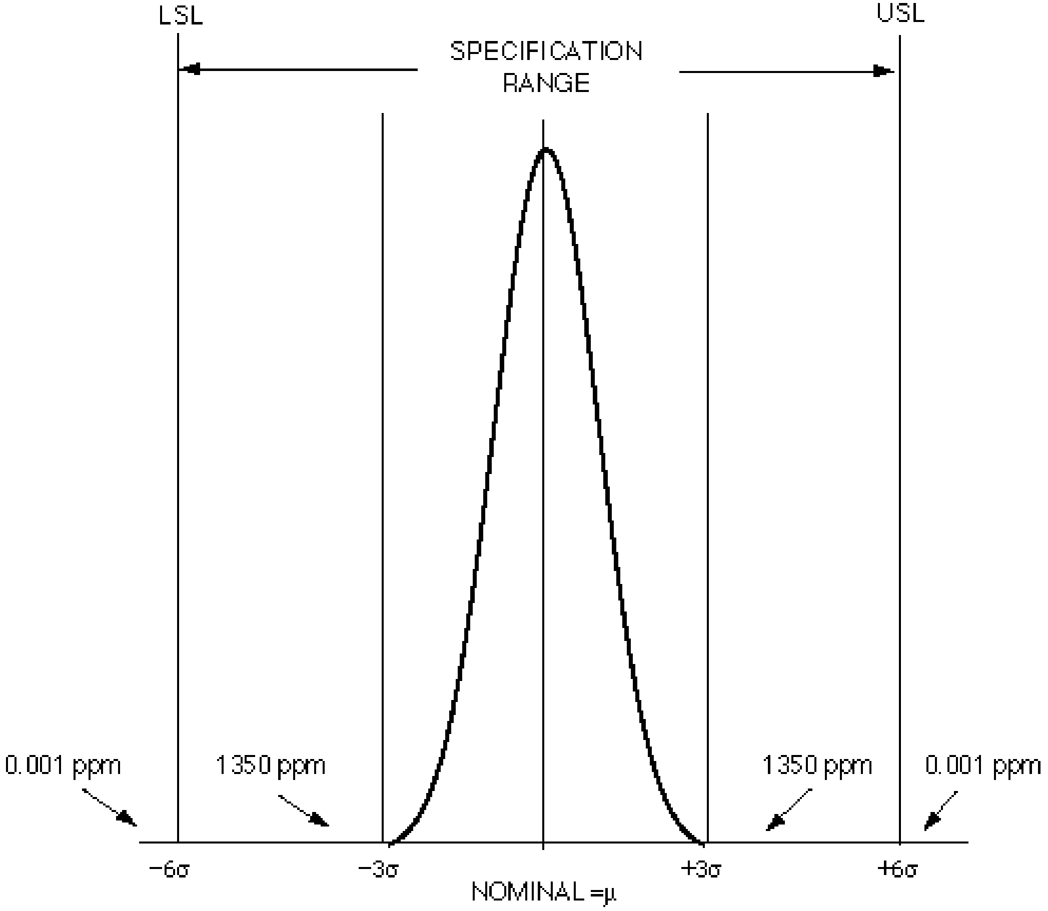
- Basic Requirement: nearest specification limit at least six sigma from process mean



$$C_p = \frac{USL - LSL}{6\sigma}$$

- **6-Sigma requirement equivalent to $C_p > 2$**

Motorola 6-Sigma Program



If process ~ normal,

6-Sigma requirement equivalent to two parts per billion nonconforming

http://www.qualitydigest.com/dec97/html/motsix.html

Process Capability Index, C_{pk}

- Accounts for bias as well as variability
- Still assumes process tolerance is $\mu \pm 3\sigma$
- $C_{pk} = C_p (1 - k)$, where k is the ratio of the bias to the specification half-width

  $k = | \tau - \mu | / \Delta$; $\Delta = (USL - LSL) / 2$

- Taguchi: easier to reduce bias than variability
  - Always reduce variability first, then re-center

Median C_{pk} in U.S. industry for key components is 2.09
Industry Week, July 17, 1995

Exercise: variability

- Mark 6 cm lengths
  - Use knuckle of thumb to tip of thumb ~3 cm
- Cut and measure to nearest 0.1 cm
- Record length and sequential index
- Construct frequency distribution
- Calculate statistics
  - Mean, mode, median, variance, range

Exercise: subgroups

- Construct subgroups for length data
- Construct frequency distribution
- Calculate statistics
  - Mean, mode, median, variance, range
- Discuss statistics of subgroup data
- Note that $\sigma_{XBAR} = \sigma_{X}$ / SQRT (n)

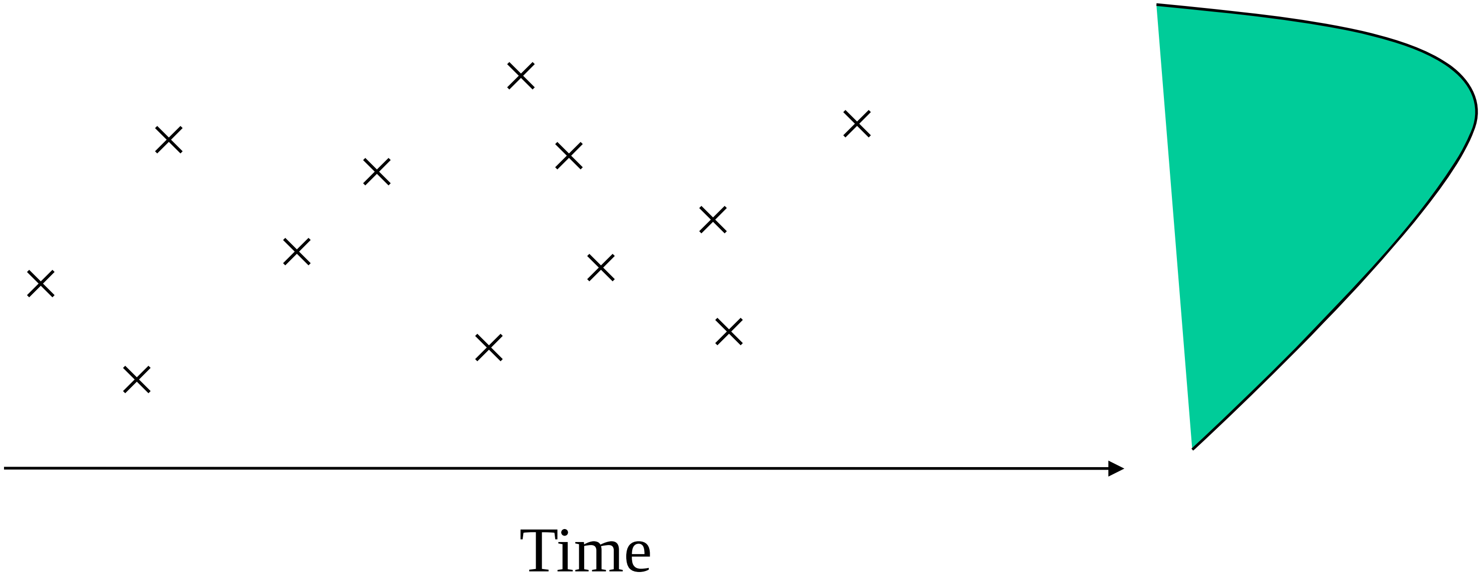
Control Charts

- A tool to determine if process is in control
- If not, find and remove "assignable causes" of process variation
- If in control, leave process alone
- **Type I error**: conclude that process has changed when in fact it has not (false alarm)
- **Type II error**: fail to detect process changes

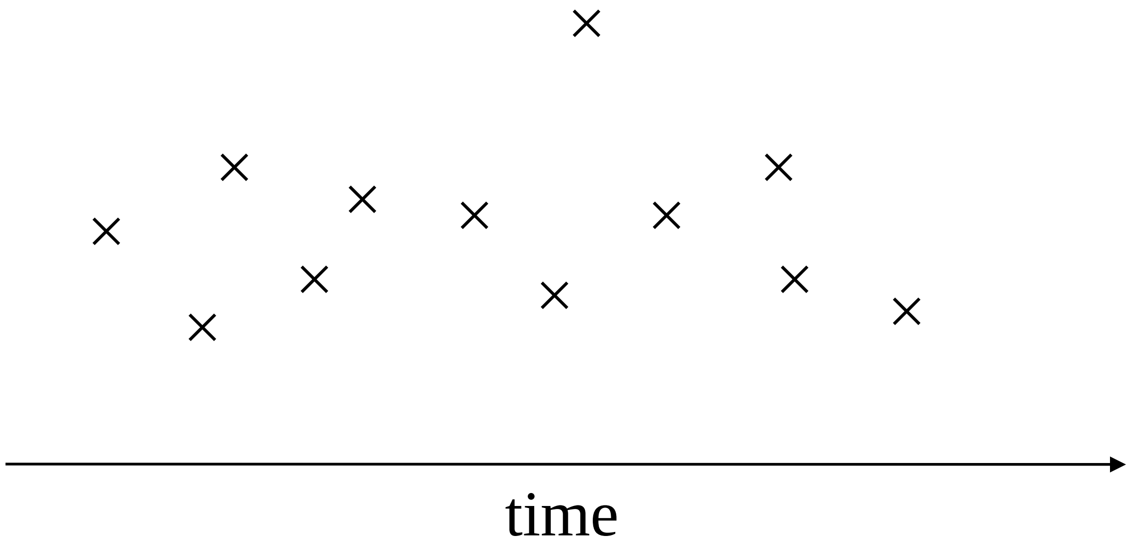
See Fig. 11.2

Control Charts

- The primary difference between control charts and frequency distributions is that control charts preserve the time order of observations

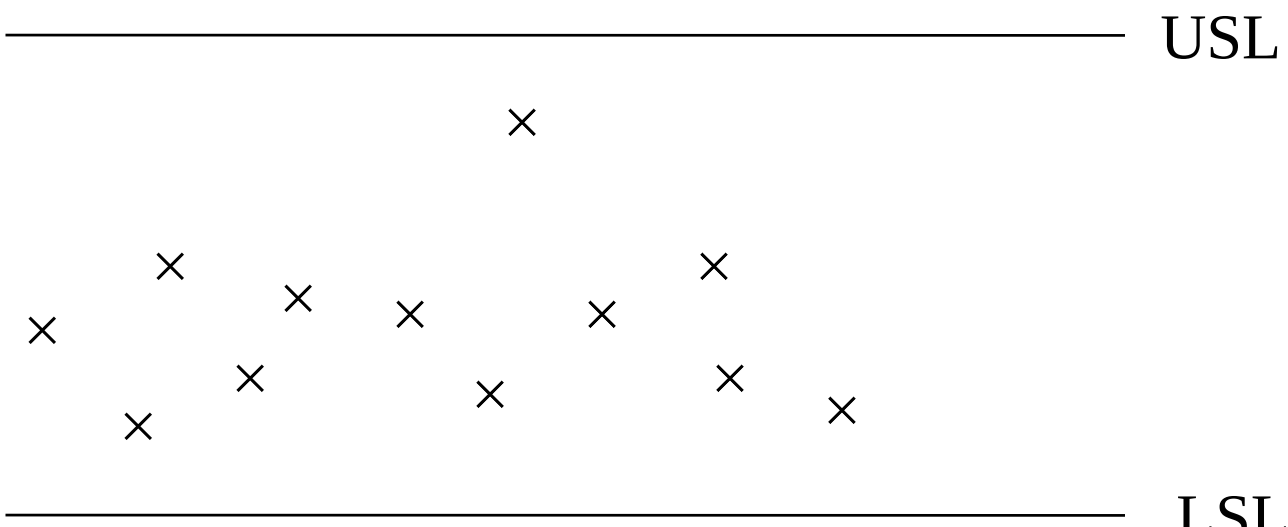


Control Charts



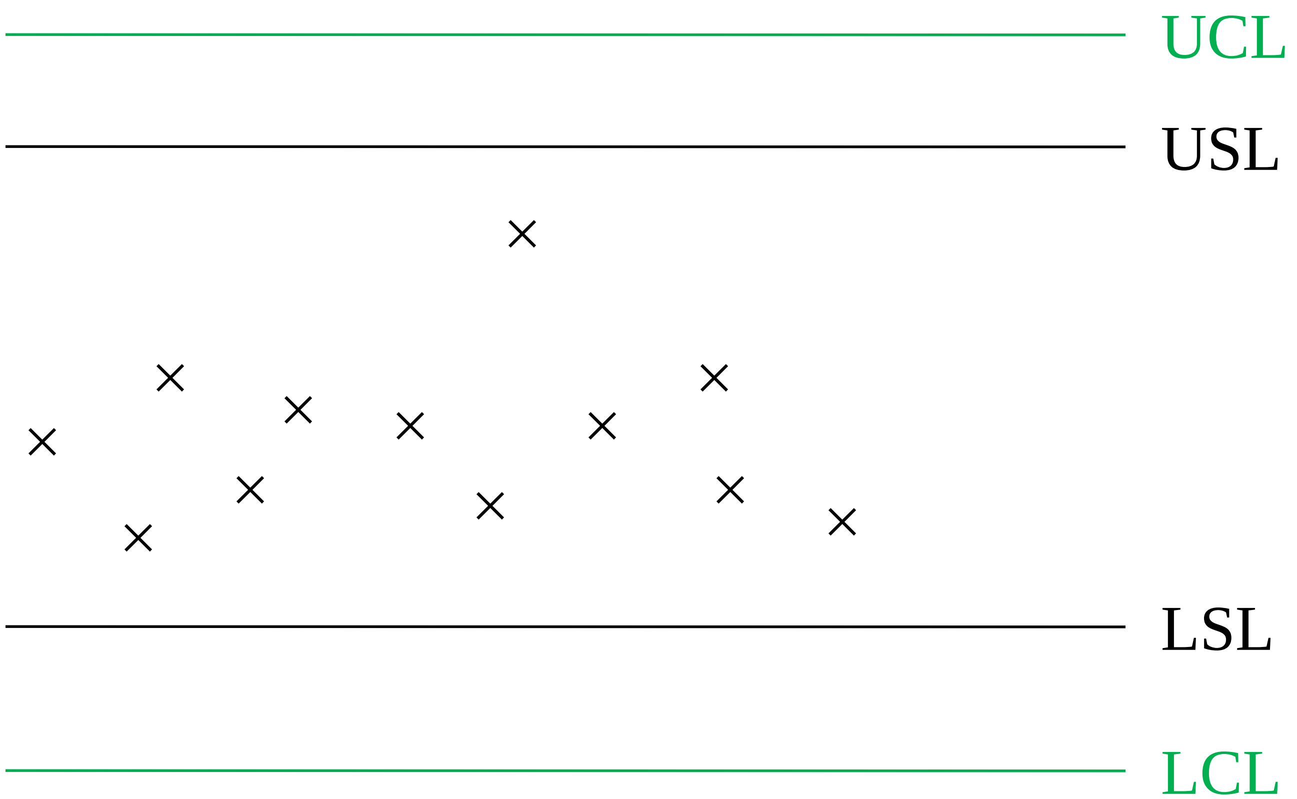
Is this process in control?

Control Charts



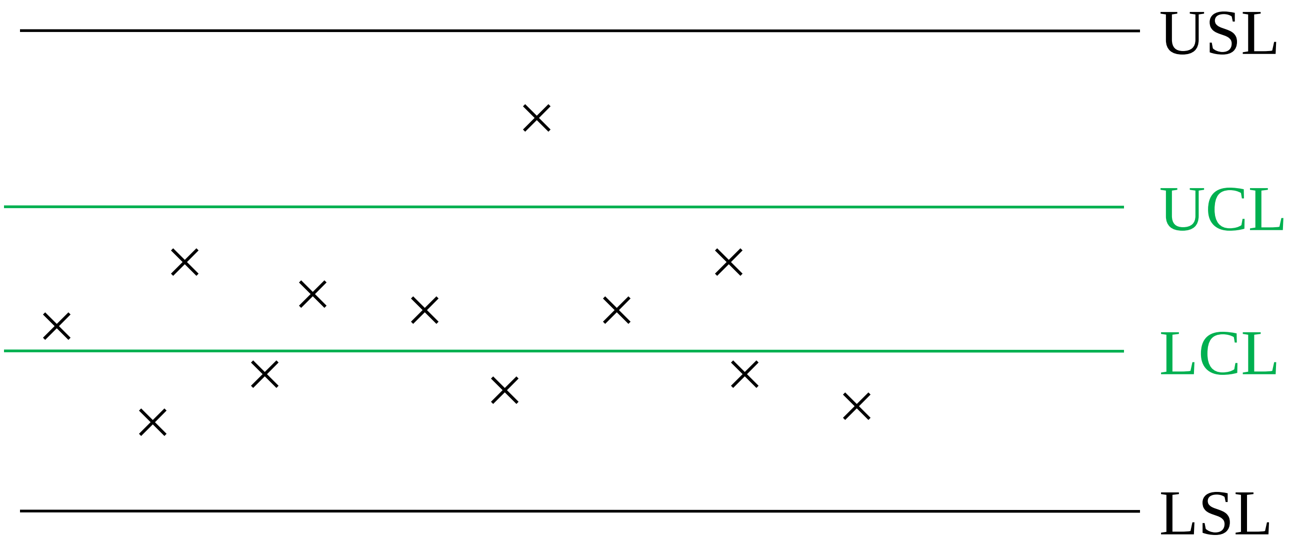
Is this process in control?

Control Charts



Is this process in control?

Control Charts



Is this process in control?

Control Chart Diagnostics

- A process may be out of control if:
  - One point plots outside 3-sigma control limits
  - Two of three consecutive points plot outside 2-sigma "warning limits"
  - Four of five consecutive points plot outside 1-sigma limits
  - Seven or more points form a "run-up" or "run-down"
  - Eight or more consecutive points plot on one side of the center line
  - Any unusual or apparently non-random pattern

Source: Western Electric Handbook

Lack of Control: Average

Subgroup	A	B	C	D	XBAR	R
1	10	10	12	12	11	2
2	10	10	12	12	11	2
3	5	5	7	7	6	2
4	5	5	7	7	6	2

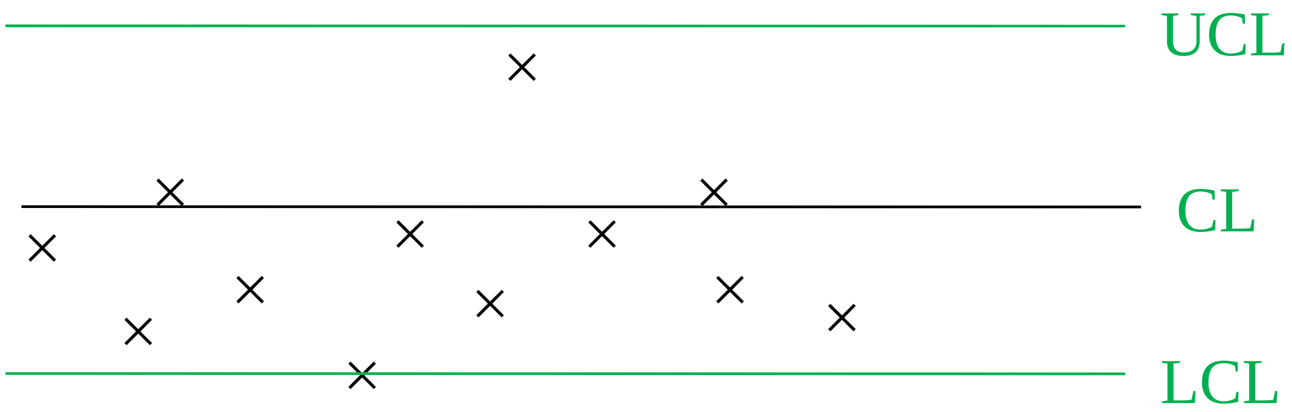
Lack of control may involve average, variability, or both

Lack of Control: Variability

Subgroup	A	B	C	D	XBAR	R
1	10	10	12	12	11	2
2	10	10	12	12	11	2
3	8	8	14	14	11	6
4	8	8	14	14	11	6

Need two control charts: XBAR and R

XBAR Charts



- Plot subgroup means to take advantage of CLT
- CL = mean subgroup average: E(XBAR) = μ
- UCL = XBAR + $3\sigma_{XBAR}$ = XBAR + $3\sigma_X$ / SQRT(n)
- LCL = XBAR – $3\sigma_{XBAR}$ = XBAR – $3\sigma_X$ / SQRT(n)

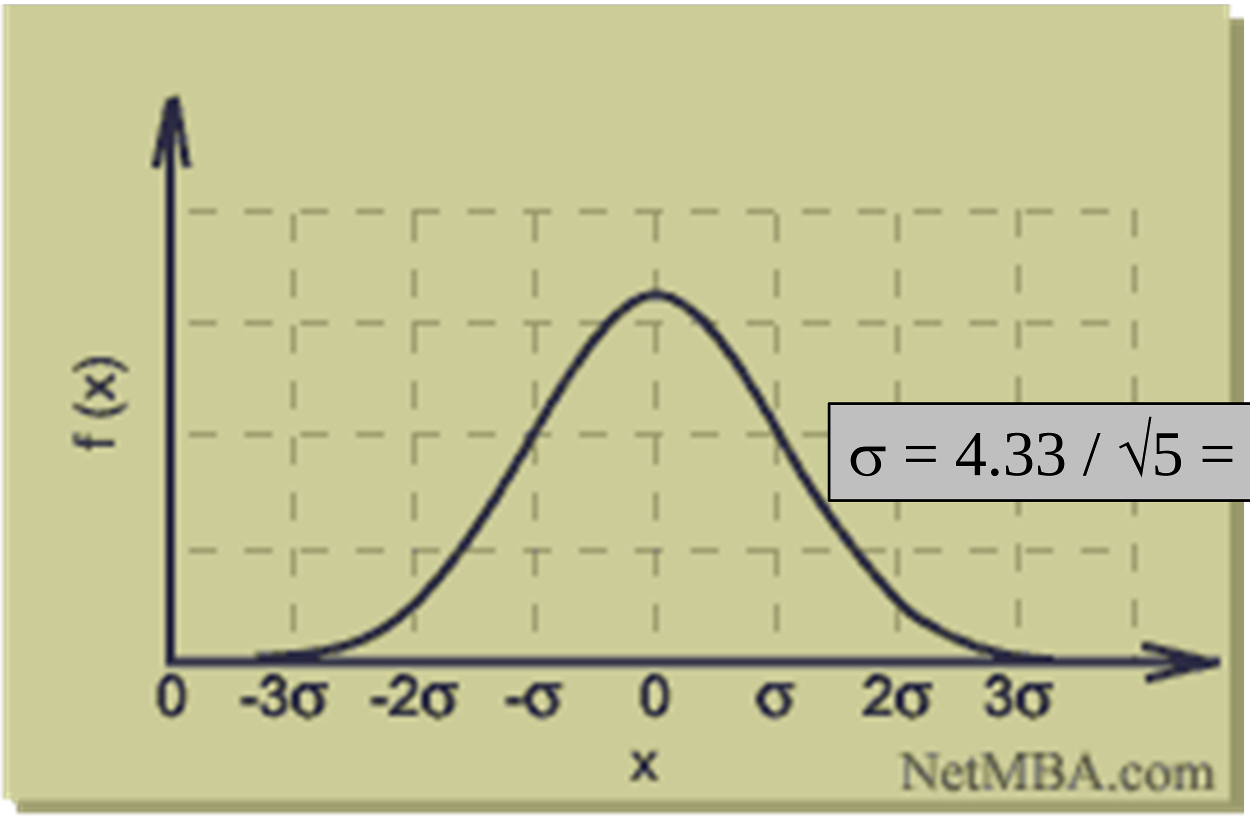
Numerical Example

- Problem 11.5. n = 10, XBAR = 0.0250 in., and RBAR = 0.0020 in.
- Specify the control limits for XBAR chart
  - UCL = XBAR + A_2 RBAR (11.5)
  - UCL = 0.0250 + 0.308 (0.0020) Table 11.2
  - UCL = 0.0256
  - LCL = XBAR - A_2 RBAR = 0.0244
- Suggested practice problems: 11.6, 11.7

Insights into Table 11.3

- Ref: pp. 330-333 and data in problem 11.6
  - Grand average = 46.04, $\sigma = 4.33$
- RBAR = 7.5 (11.2)
- $\sigma_{est.}$ = RBAR / d_2 = 7.5 / 2.326 = 3.22 (11.3)
- $3\sigma_{XBAR}$ = 3RBAR / (d_2 $\sqrt{n}$) = 4.33 (11.4)
- UCL_{XBAR} = 46.04 + A_2 7.5 = 50.37 (11.5)
- UCL_{XBAR} = 46.04 + $3\sigma/\sqrt{n}$ = 51.85 (F.P.)
  - in this example, use of tables increased probability of Type I error from 0.0027 to 0.0258

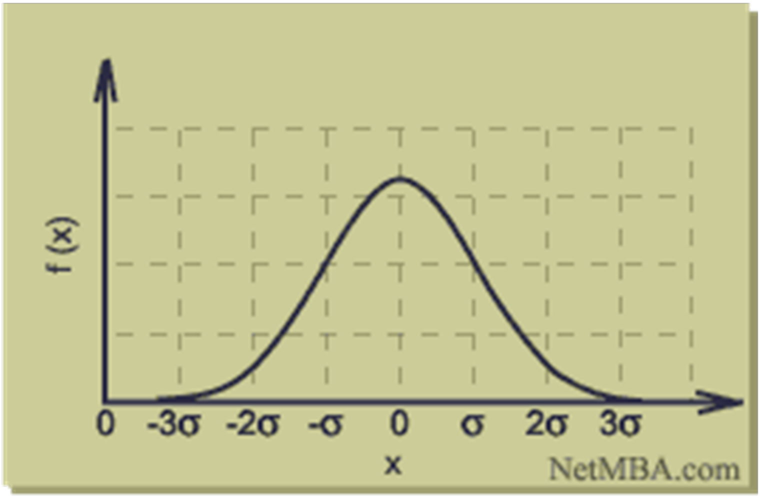
Calculating P(Type I error)



$\sigma = 4.33 / \sqrt{5} = 1.94$

LCL | 46.04 | UCL

Calculating P(Type I error)



XBAR = 46.04

$\sigma_{XBAR} = 4.33 / \sqrt{5} = 1.94$

P(Type I error) = P (X < LCL) + P (X > UCL)

P (X > UCL) = P (Z > (50.37 – 46.04) / 1.94 = P (Z > 2.23) = .0129

P(Type I error) = 2 P (X > UCL) = .0258

Ref: Table D.3

Starting a control chart

- What decisions are required?
  - Choice of variable
  - How to make measurements
  - Who will make measurements
  - What to record (time, comments, etc)
  - How to group measurements into subgroups
  - Who can take action

Subgroups

- In general, we want minimum variation within subgroups, with maximum variation between subgroups
- In manufacturing, subgroups typically based on time of production
- What are other possibilities?
  - Within operator, within machine, etc.

Size of subgroups

- Large enough to
  - use normal sampling theory
  - Make type II errors unlikely
- Small enough to
  - Keep measurement costs low
  - Reduce variability within subgroups
- Shewhart suggested 4 as an ideal size

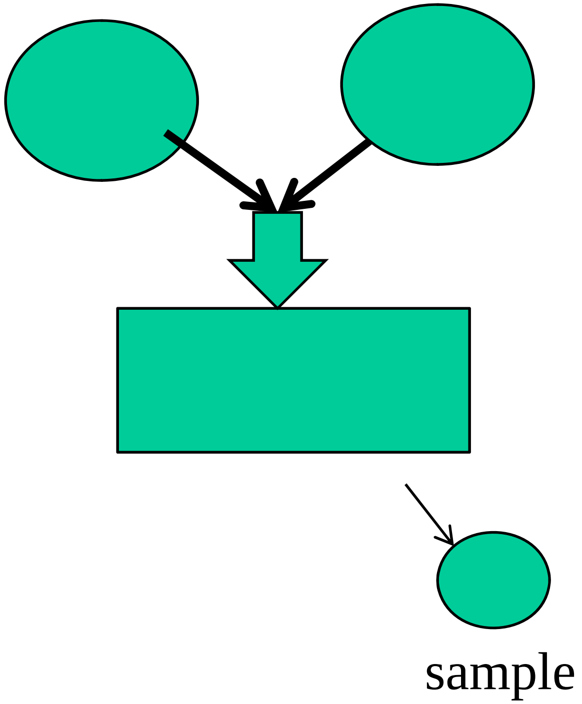
Subgroups

- Process: μ=10; σ=1.9
- Subgroup: n = 4
- UCL = 10 + 3σ/2 = 12.9
- UCL = 10 – 3σ/2 = 7.2
- By row: out of control
- By column: no problem
- Construct subgroups based on knowledge of the process
- Special causes of variation may be hidden by improper subgrouping

	A	B	C	D	Avg.
1	10	11	12	12	11.3
2	11	12	12	11	11.5
3	6	7	8	7	7.0
4	9	10	11	9	9.8
Avg.	9	10	10.8	9.8	

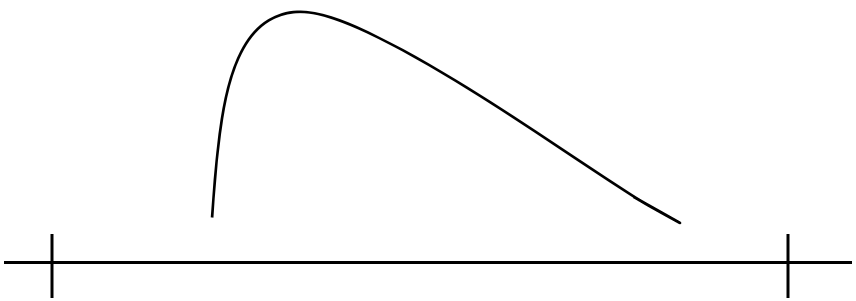
Example

- Two machines produce parts at 100 units per hour each
- Parts outside specs. are be scrapped
- Items from both machines are discharged into a single tote box
- Inspector selects a subgroup of 5 parts every one-half hour.
- Adjustments to both machines are made only on the approval of the inspector
- How could we improve this process?



Process Capability

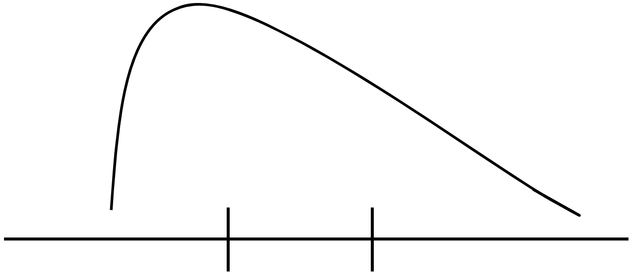
- 6σ a typical estimate of process capability
- What if 6 σ < USL – LSL?



- Reduce inspection
- Tighten specs to help marketing or downstream production

Process Capability

- What if $6\sigma > USL - LSL$?



- Reexamine spec limits
- Look for ways to reduce variability
- 100% inspection
- Fundamentally change the process
- Give up the business

11.2.3 Control Charts for Attributes

- Many quality characteristics can be observed only as attributes
  - Hit / miss
  - Conforming / not conforming
  - Ready / not ready
- We can monitor the proportion of observations falling into one of two classes using a *p* chart

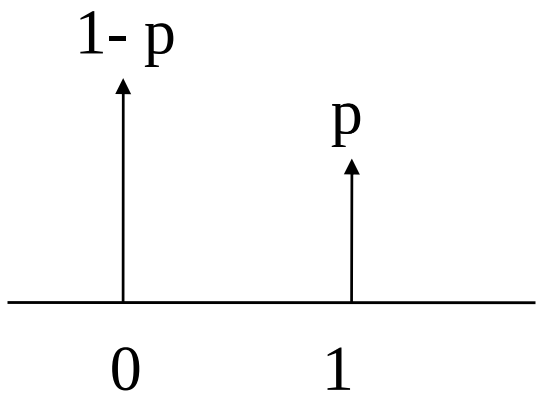
Bernoulli Trials

- An experiment with only two outcomes
- Named for the Swiss theologian and mathematician who described these experiments in his book, *The Art of Conjecture*, published eight years after his death in 1713

James Bernoulli
1654 - 1705

Bernoulli Distribution

- Consider an experiment with outcomes 0 and 1 with probabilities (1-p) and p, respectively
- This forms a complete probability distribution, called the Bernoulli distribution



$$\mu = \Sigma\, x\, p(x) = 0\,(1-p) + 1\, p = p$$

$$\sigma^2 = E(x^2) - [E(x)]^2$$

$$= 0^2(1-p) + 1^2(p) - p^2$$

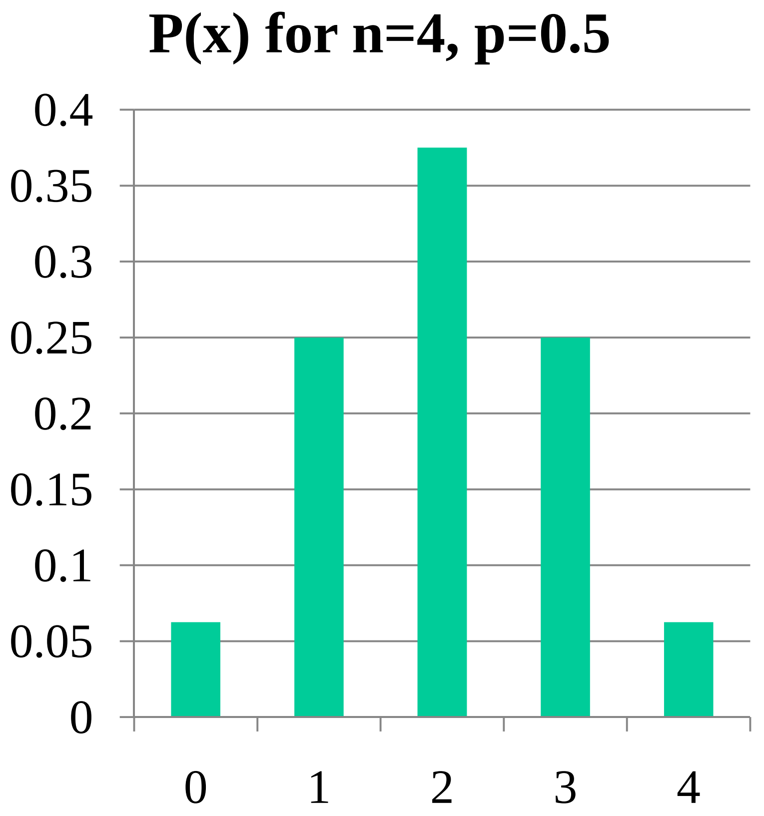
$$= p - p^2 = p\,(1-p)$$

Binomial Distribution

- Consider a series of Bernoulli trials, sometimes called a Bernoulli process, more commonly called a binomial experiment
  - The experiment consists of *n* identical trials
  - Each trial has one of two outcomes: 0 or 1
  - The probability of a "1", denoted *p*, is constant over all trials (i.e., each trial is independent)
  - The RV, X, is the number of "1"s in n trials
- The binomial distribution gives the probability of exactly X "1"s in *n* trials

Binomial Distribution

- RV X is number of successes in *n* trials
- $P(x) = {}_nC_x\, p^x\, (1-p)^{n-x}$
- $E(X) = n\,p$
- $\sigma^2 = n\,p\,(1-p)$



Proportion

- Consider a new RV, X/n, the proportion, or fraction of successes in n trials
- $E(X/n) = 1/n\, E(X) = n\, p \,/\, n = p$
- $Var(X/n) = 1/n^2\, Var\,(X) = n\, p(1\text{-}p)/n^2 = p\,(1\text{-}p)\,/\,n$
- Control Limits for proportions
  - $UCL = E(X/n) + 2\,\sigma_{X/n} = p + 2\sqrt{[p\,(1\text{-}p)\,/\,n]}$ (11.11)
  - $LCL = E(X/n) - 2\,\sigma_{X/n} = p - 2\sqrt{[p\,(1\text{-}p)\,/\,n]}$ (11.12)

Example

- A certain product is 100% inspected as it is manufactured.
- Calculate 2σ control limits using the data provided
- What values are out of control?

Hour	# units	defects	Frac.
1	48	5	.104
2	36	5	.139
3	50	0	0
4	47	5	.106
5	48	0	0
6	54	3	.056
7	50	0	0
8	42	1	.024
9	32	5	.156
10	40	2	.050
11	47	2	.043
12	47	4	.085
13	46	1	.022
14	46	0	0
15	48	3	.063
16	39	0	0

Example

- p_{est} = .053; n = 16
- $\mu_p = p = .053$
- $\sigma^2 = p(1-p) / n = .00313$
- LCL = 0
- UCL = 0.165
- No values out of control (i.e., outside 2σ limits)

Hour	# units	defects	Frac.
1	48	5	.104
2	36	5	.139
3	50	0	0
4	47	5	.106
5	48	0	0
6	54	3	.056
7	50	0	0
8	42	1	.024
9	32	5	.156
10	40	2	.050
11	47	2	.043
12	47	4	.085
13	46	1	.022
14	46	0	0
15	48	3	.063
16	39	0	0

Another Example

- **Straight average**
- p_{est} = .109
- $\sigma^2 = p(1\text{-}p) / n = .019$
- UCL = .387

Lot	# units	defects	Frac.
1	1200	18	.015
2	750	40	.053
3	150	26	.173
4	75	15	.200
5	225	23	.102

- **Weighted average**
- p_{est} = .051
- $\sigma^2 = p(1\text{-}p) / n = .010$
- UCL = .247

Use weighted average when sample size varies significantly from lot to lot

Discussion

- From a statistical point of view, variables are superior to attributes
- Actual measurement of a few items may provide more information that classification of many items
- Classification, although easier, discards information

✓ When possible, measure the quality characteristic and construct XBAR chart rather than *p*-chart