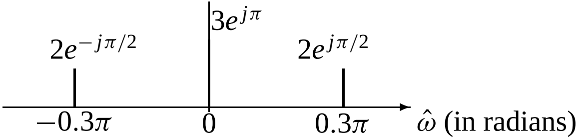
PROBLEM:

A discrete-time signal $x[n]$ has the two-sided spectrum representation shown below.



(a) Write an equation for $x[n]$. Make sure to express $x[n]$ as a real-valued signal.

(b) Determine the formula for the output signal $y[n]$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

Part A

$$\begin{aligned} x[n] &= 3e^{j\pi}e^{j0n} + 2e^{j\pi/2}e^{j0.3\pi n} + 2e^{-j\pi/2}e^{-j0.3\pi n} \\ &= \boxed{-3 + 4\cos(0.3\pi n + \pi/2)} \end{aligned}$$

Part B

Nine-point averaging filter implies that

$$y[n] = \frac{1}{9}\Big(x[n-4]+x[n-3]+x[n-2]+x[n-1]+x[n]+x[n+1]+x[n+2]+x[n+3]+x[n+4]\Big)$$

which means

$$h[n] = \frac{1}{9}\Big(\delta[n-4]+\delta[n-3]+\delta[n-2]+\delta[n-1]+\delta[n]+\delta[n+1]+\delta[n+2]+\delta[n+3]+\delta[n+4]\Big).$$

The corresponding frequency response is given by

$$\begin{aligned} \mathcal{H}(\hat\omega) &= \tfrac{1}{9}\Big(e^{-j4\hat\omega} + e^{-j3\hat\omega} + e^{-j2\hat\omega} + e^{-j\hat\omega} + 1 + e^{j\hat\omega} + e^{j2\hat\omega} + e^{j3\hat\omega} + e^{j4\hat\omega}\Big) \\ &= \tfrac{1}{9}\Big(1 + 2\cos(\hat\omega) + 2\cos(2\hat\omega) + 2\cos(3\hat\omega) + 2\cos(4\hat\omega)\Big) \\ \mathcal{H}(0) &= \tfrac{1}{9}(1+2+2+2+2) = 1 \\ \mathcal{H}(0.3\pi) &= \tfrac{1}{9}\Big(1 + 2\cos(0.3\pi) + 2\cos(0.6\pi) + 2\cos(0.9\pi) + 2\cos(1.2\pi)\Big) \\ &= \tfrac{1}{9}\Big(1 + 1.1755 - 0.6180 - 1.9021 - 1.6180\Big) = -0.2181 \end{aligned}$$

$$y[n] = -3(1) + 4(-0.2181)\cos(0.3\pi n + \pi/2) = \boxed{-3 + 0.8724\cos(0.3\pi n - \pi/2)}$$

If the nine-point averaging filter is constrained to be *causal*:

$$y[n] = \frac{1}{9}\Big(x[n]+x[n-1]+x[n-2]+x[n-3]+x[n-4]+x[n-5]+x[n-6]+x[n-7]+x[n-8]\Big)$$

Then the frequency response contains an additional phase term:

$$\mathcal{H}(\hat\omega) = \tfrac{1}{9}\Big(1 + 2\cos(\hat\omega) + 2\cos(2\hat\omega) + 2\cos(3\hat\omega) + 2\cos(4\hat\omega)\Big)e^{-j4\hat\omega}$$

and $y[n]$ will be delayed by 4, because the filter's impulse response is shifted right by 4.

$$y[n] = -3 + 0.8724\cos(0.3\pi(n-4) - 0.5\pi) = \boxed{-3 + 0.8724\cos(0.3\pi n + 0.3\pi)}$$